Prospects for evaluation of the resonance electrocouplings from first principles QCD

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Jefferson Lab

Hall B/Theory Meeting
June 6th 2017

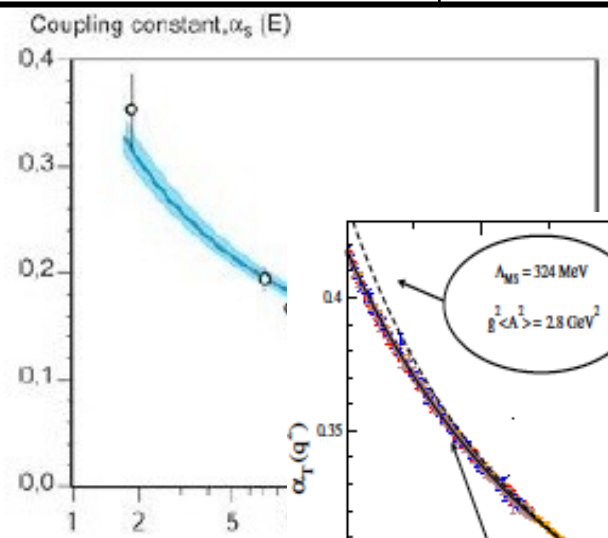
Outline

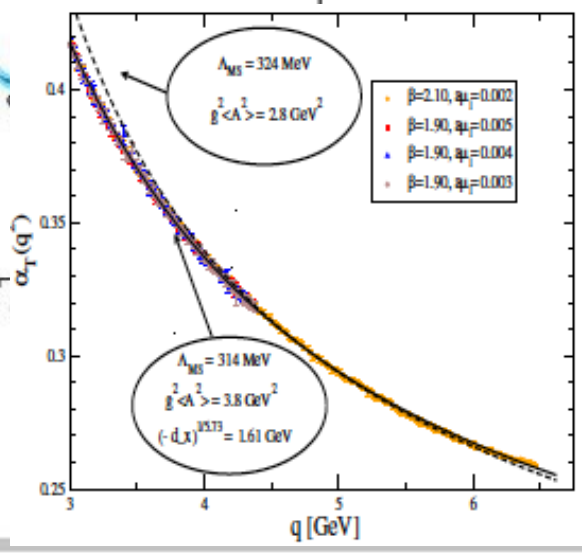
- Lattice QCD
- Resonances on the lattice
- Developments for Mesons
- Challenges for Baryons
- Prospects

Quantum Chromodynamics (QCD)

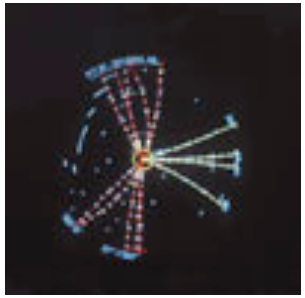
QED	QCD
Photon, γ	Gluons, G
Charged particles, e, μ, u, d,…	Quarks: u, d, s, c, b, t
Photon is *neutral*	Gluons carry *color charge* Theory is *non-Abelian*
α_e *=1/137*	α_s *~O(1)*

Infrared Slavery
Lattice QCD





Asymptotic Freedom
Pert. Theory



Gluons in spectrum!

Gluons in three-jet event

Lattice QCD

Observables in lattice QCD are then expressed in terms of the path integral as

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \prod_n d\psi(n) \prod_n d\bar{\psi}(n) \mathcal{O}(U, \psi, \bar{\psi}) e^{-\left(S_G[U] + S_F[U,\psi,\bar{\psi}]\right)}$$

Integrate out the Grassmann variables:

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \prod_{n,\mu} dU_\mu(n) \mathcal{O}(U, G[U]) \det M[U] e^{-S_G[U]}$$

Importance Sampling

where $G(U,x,y)^{ij}_{\alpha\beta} \equiv \langle \psi^i_\alpha(x) \bar{\psi}^j_\beta(y) \rangle = M^{-1}(U)$

- Generate an ensemble of gauge configurations

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

- Calculate observable

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

Hierarchy of Computations

Capability Computing -
Gauge Generation

e.g. Cray XK6 at ORNL

$$P[U] \propto \det M[U] e^{-S_G[U]}$$

Several *V, a, T, m_π*

Capacity Computing -
Observable Calculation

e.g. cluster at JLab

$$\langle \mathcal{O} \rangle = \frac{1}{N} \sum_{n=1}^{N} \mathcal{O}(U^n, G[U^n])$$

$$\text{e.g. } C(t) = \sum_{\vec{x}} \langle N(\vec{x}, t) \bar{N}(0) \rangle$$

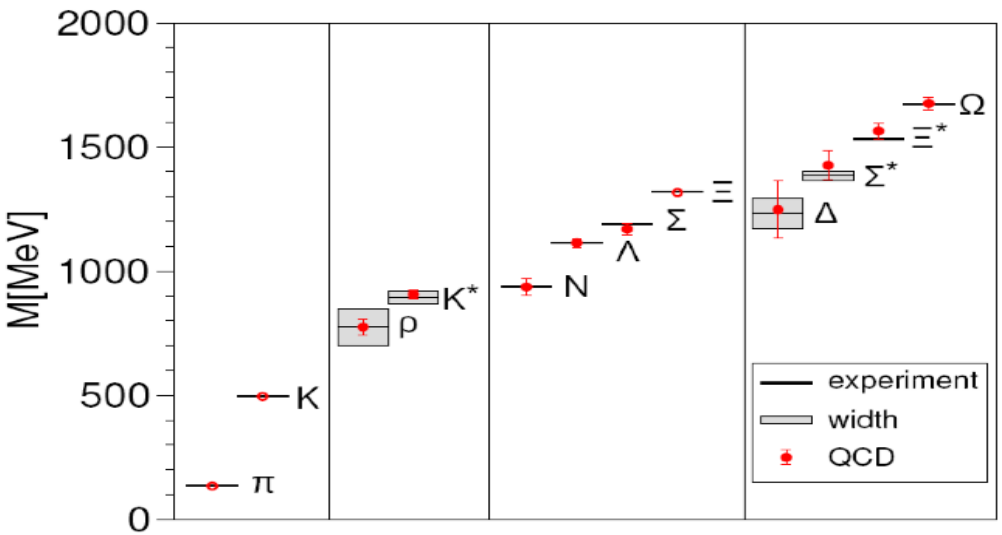
"Desktop" Computing -
Physical Parameters

e.g. Desktop

$$C(t) = \sum_n A_n e^{-E_n t}$$

$$M_N(a, m_\pi, V)$$

Low-lying Hadron Spectrum



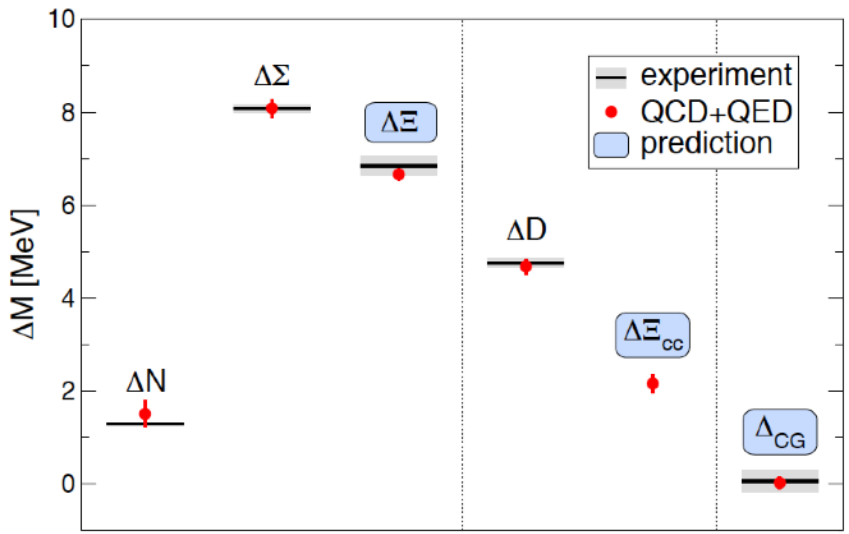
$$C(t) = \sum_n A_n e^{-E_n t}$$

Durr et al., BMW Collaboration

Science 2008

Now with electro-magnetic splittings included

Science 2015



Excited-State Spectrum

- In finite volume, energy levels are *discrete* **No cuts**

$$C_{ij}(t) = \sum_N \frac{Z_i^{N*} Z_j^N}{2E_N} e^{-E_N t} \qquad Z_i^N = \langle N \mid \bar{N}_i(0) \mid 0 \rangle$$

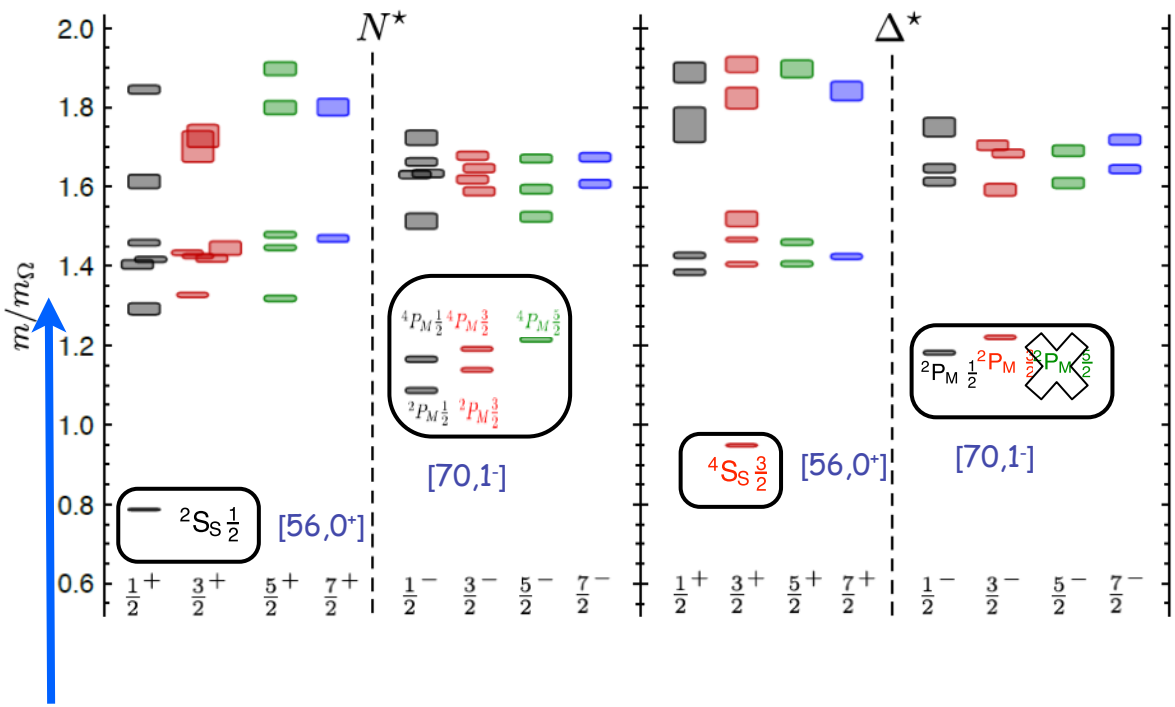
Variational Method

$$C(t)v^{(N)}(t,t_0) = \lambda_N(t,t_0)C(t_0)v^{(N)}(t,t_0).$$

$$\lambda_N(t,t_0) \to e^{-E_N(t-t_0)}(1 + \mathcal{O}(e^{-\Delta E(t-t_0)}))$$

Discrete set of energy levels

Excited Baryon Spectrum



Broad features of SU(6)xO(3) symmetry.

Counting of states consistent with NR quark model.

Inconsistent with quark-diquark picture or parity doubling.

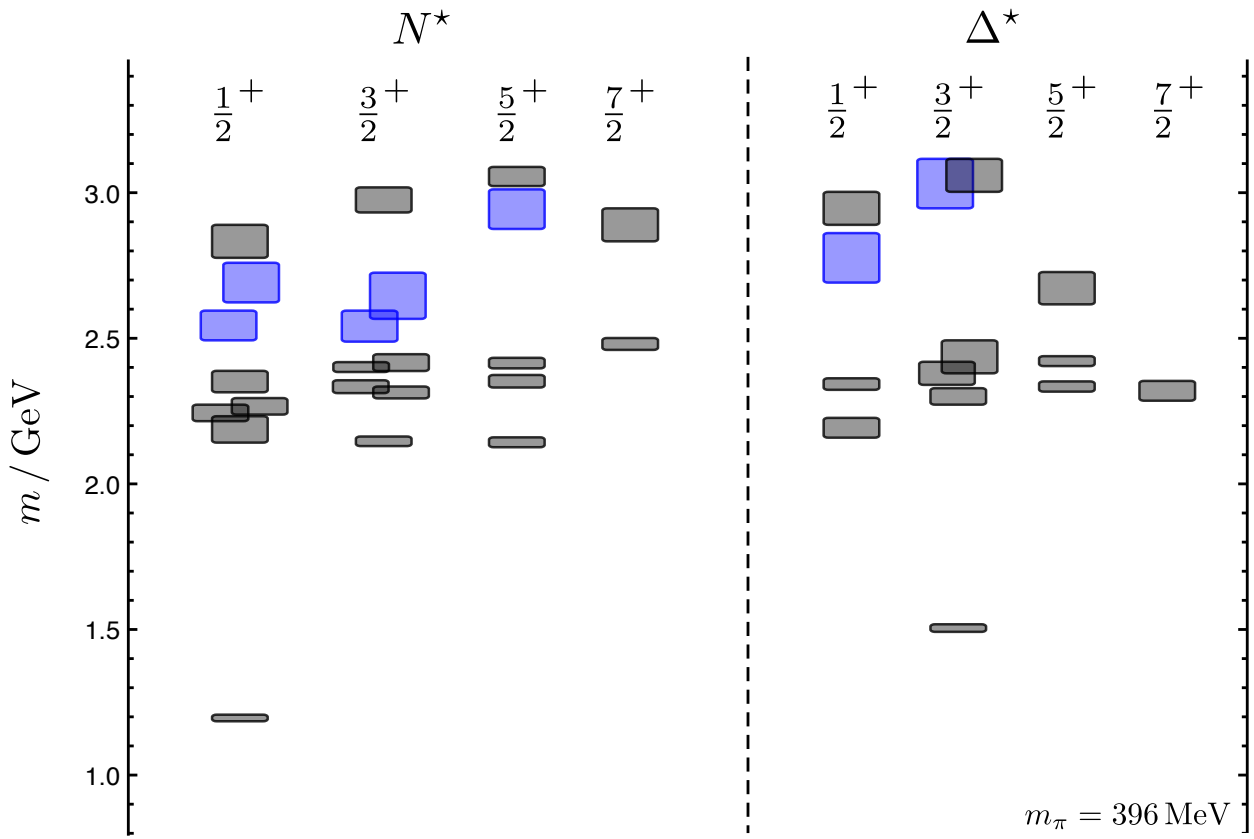
$[70, 0^+], [56, 2^+], [70, 2^+], [20, 1^+]$

R.G.Edwards et al., arXiv:1104.5152

N $^{1/2+}$ sector: need for complete basis to faithfully extract states

“Baryon Spectrum”

Dudek, Edwards, arXiv:1201.2349



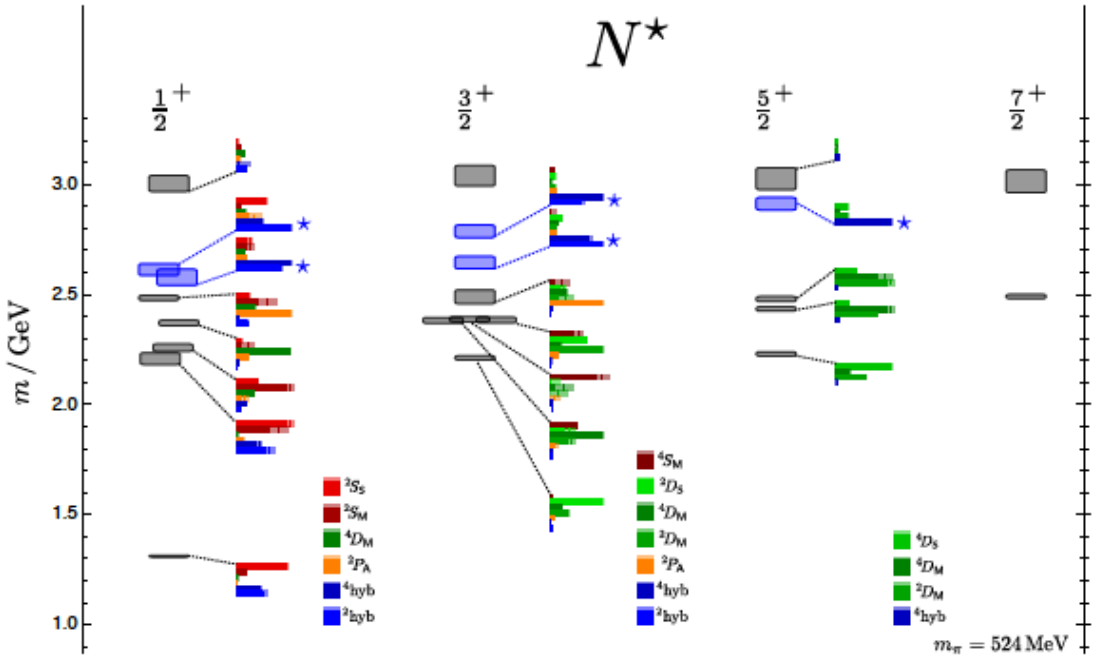
“Single-hadron approximation”

$$B = \left(\mathcal{F}_{\Sigma_F} \otimes \mathcal{S}_{\Sigma_S} \otimes \mathcal{D}_{\Sigma_D}\right)\{\psi_1\psi_2\psi_3\}$$

Resonances under strong interaction

“Luscher Method”

N*: Interpolating Operators

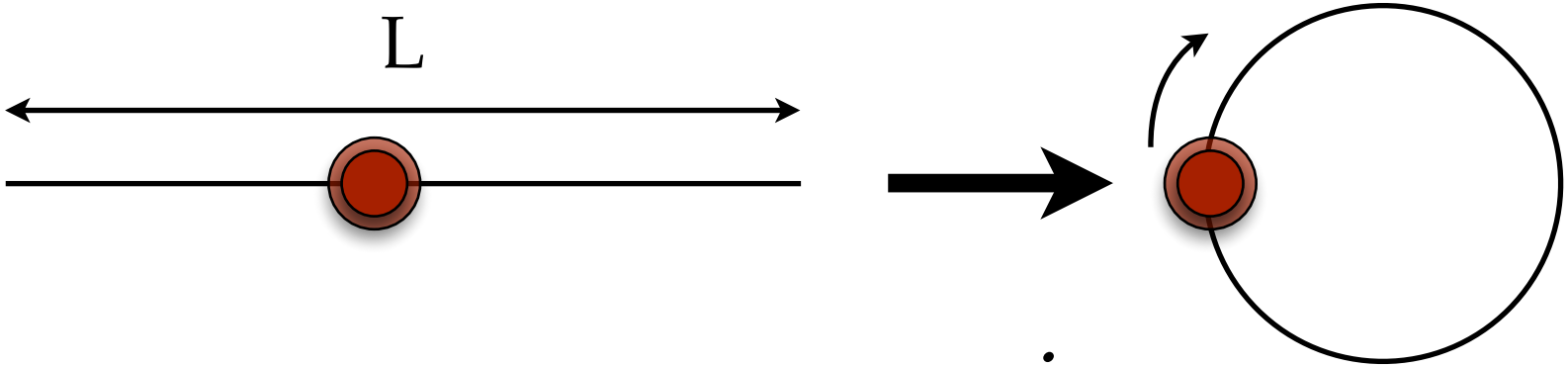


Examine overlaps onto different NR operators, i.e. containing upper components of spinors: *ground state has substantial hybrid component*

Variational method gives important *indications* of structure of hadrons

Reinventing the *quantum-mechanical* wheel

(in 1+1 dimensions)



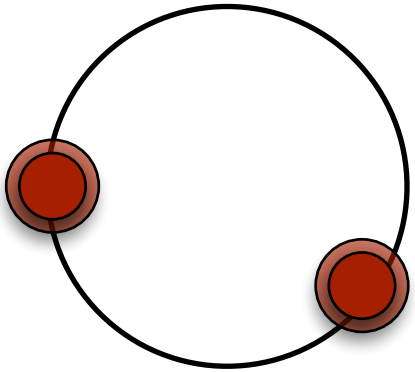
$$\phi(x) \sim e^{ipx}$$

Periodicity: $L\ p_n = 2\pi n$

Slides: thanks to Raul Briceno

Reinventing the *quantum-mechanical* wheel

Two particles:



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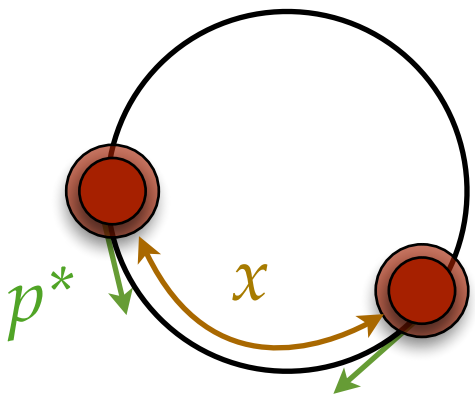

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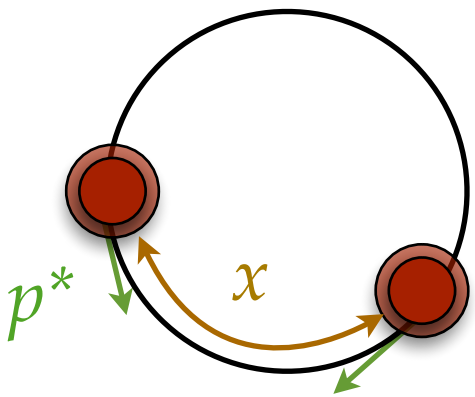


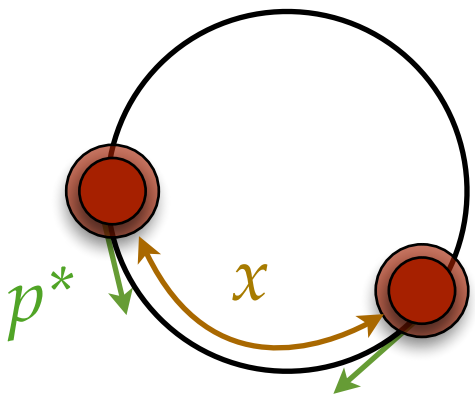
infinite volume scattering phase shift

$$\psi(x) \sim e^{ip^*|x| + i2\delta(p^*)}$$

Asymptotic wavefunction

Reinventing the *quantum-mechanical* wheel

Two particles:



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Asymptotic wavefunction

Periodicity: $$L\ p_n^* + 2\delta(p_n^*) = 2\pi n$$

Reinventing the *quantum-mechanical* wheel

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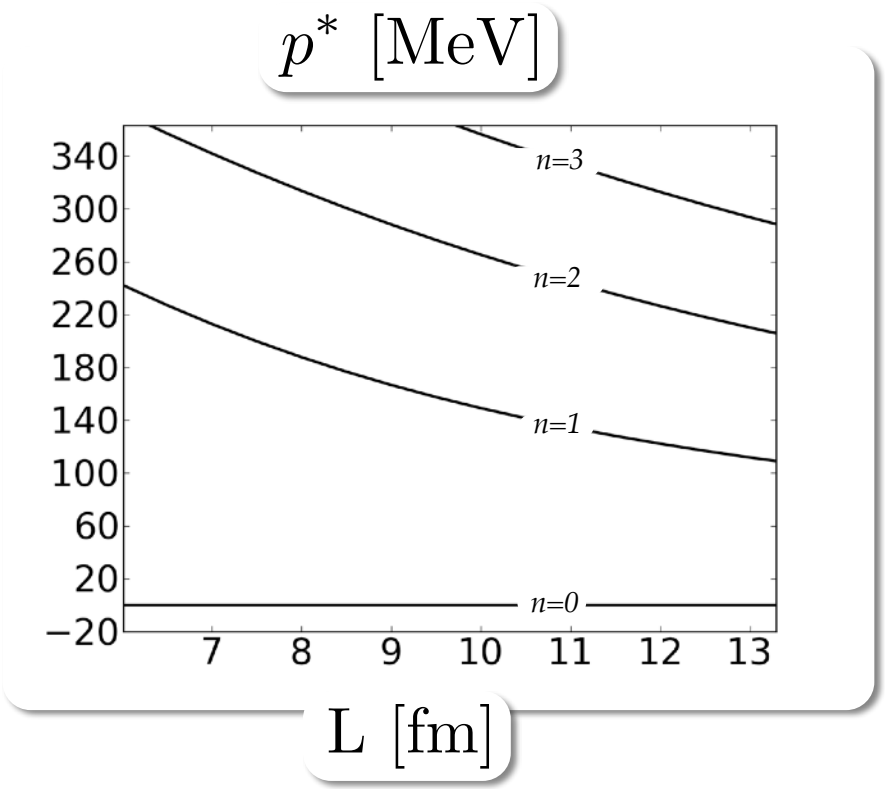

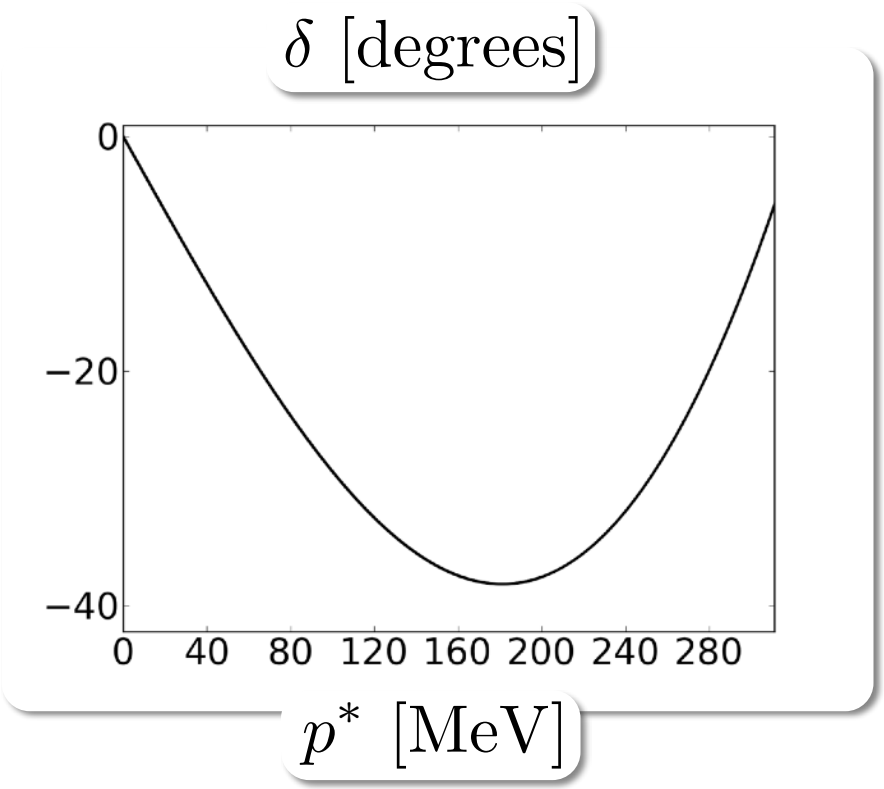

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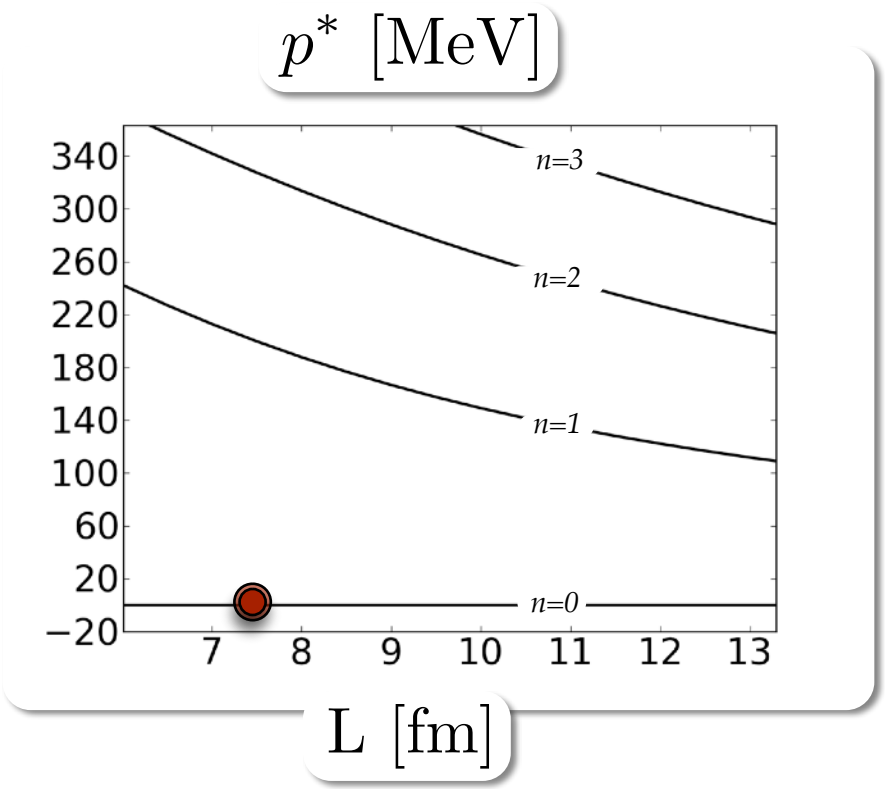

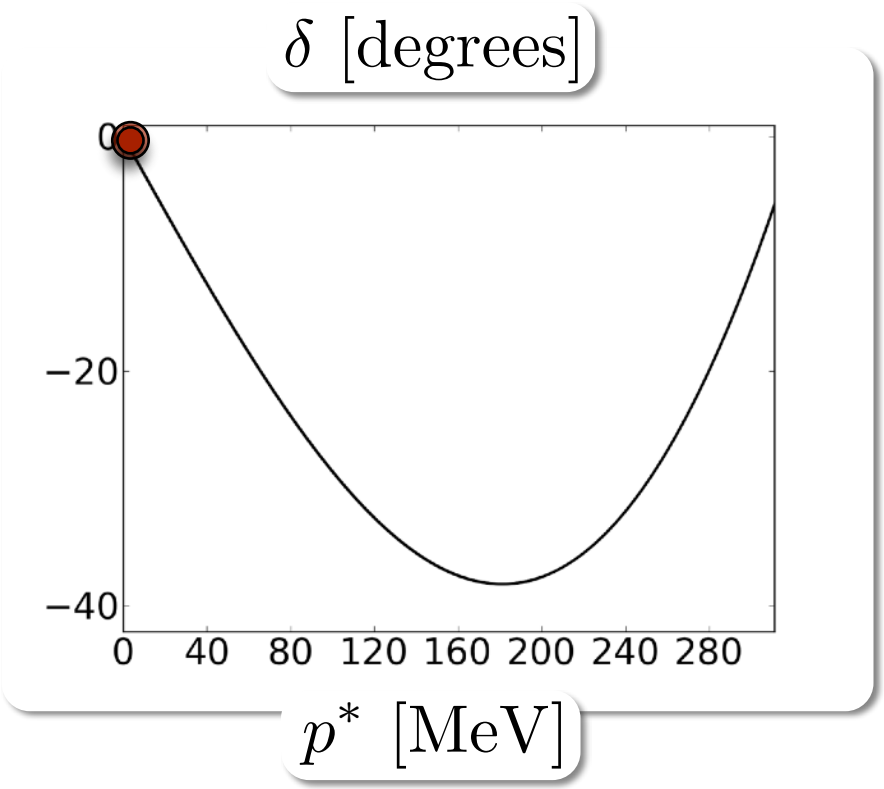

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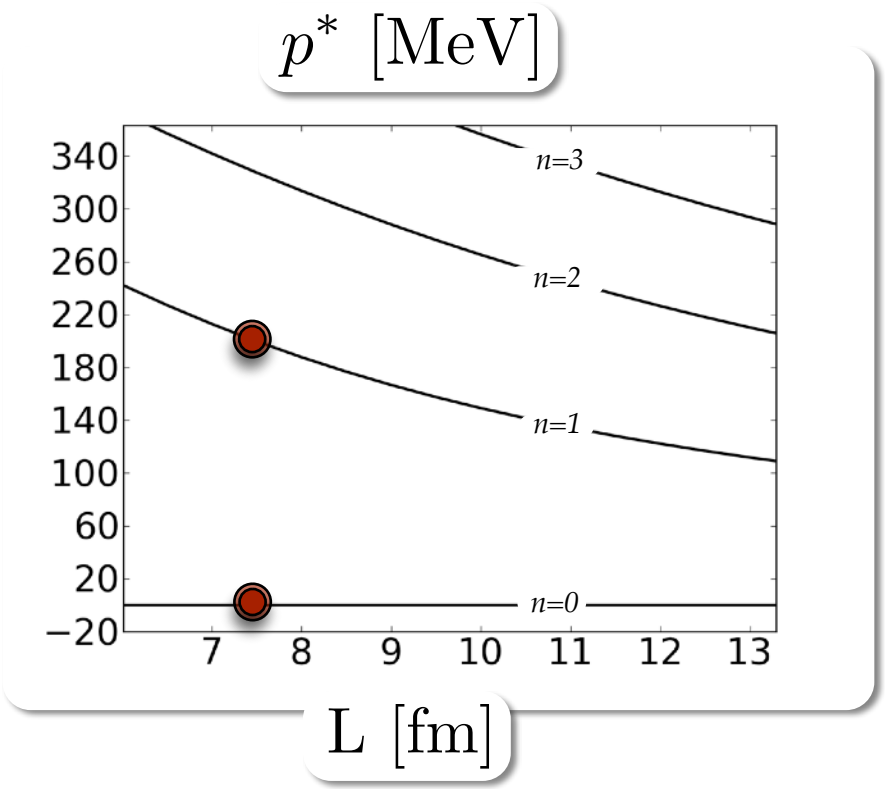

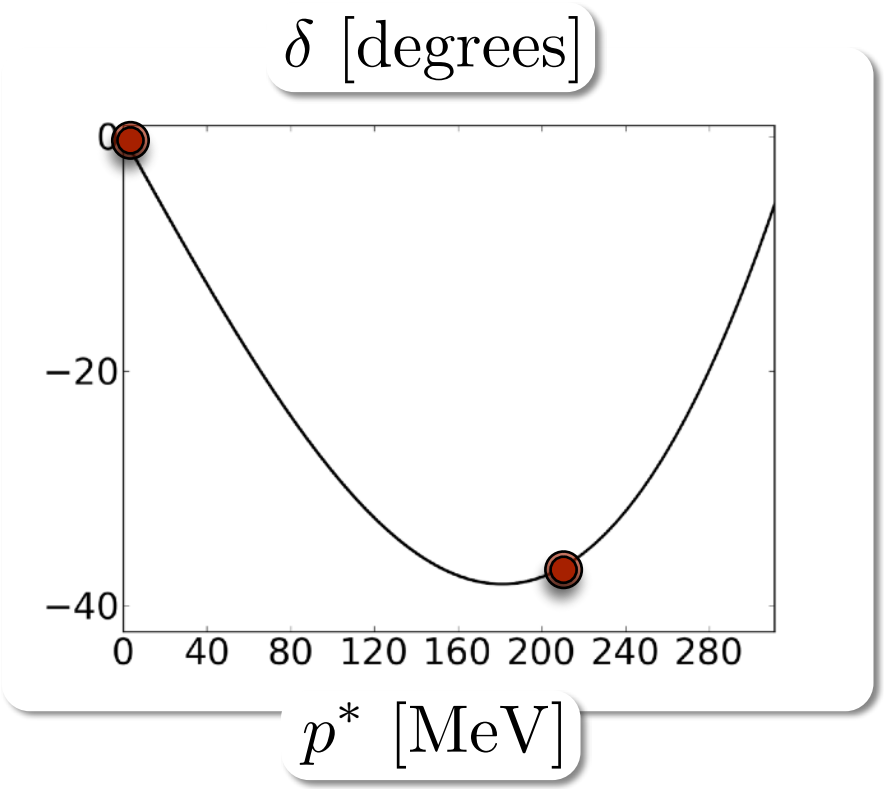

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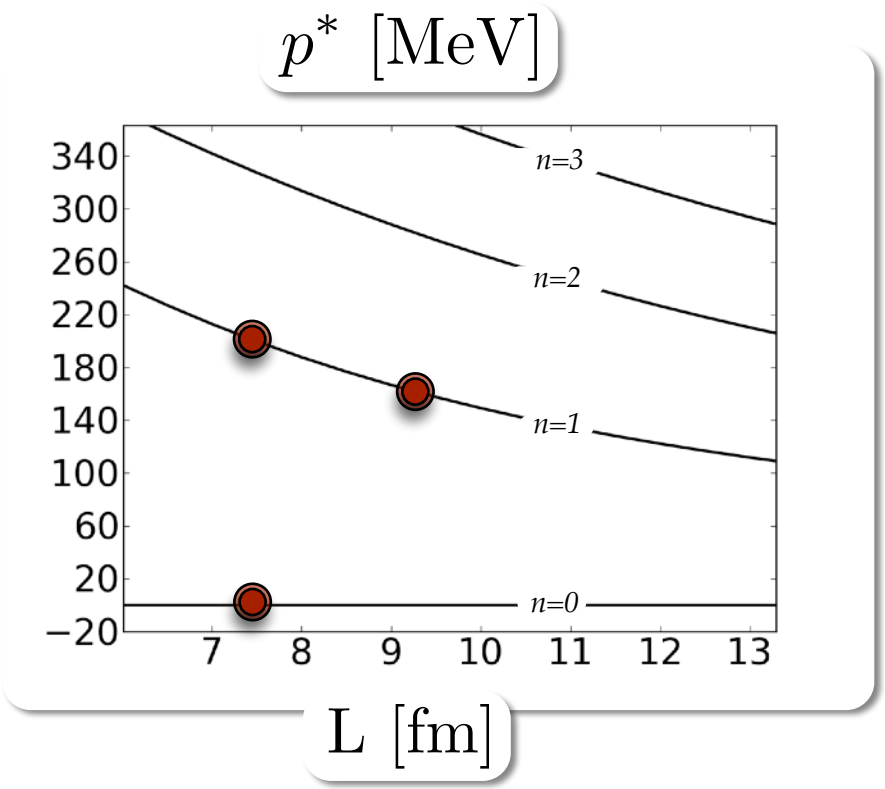

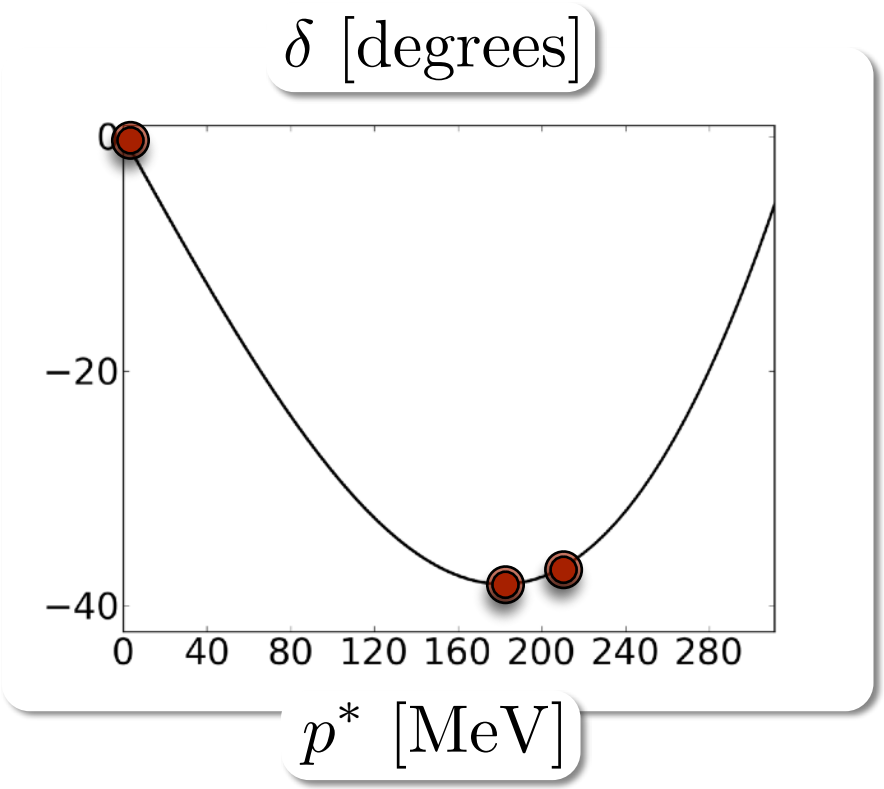

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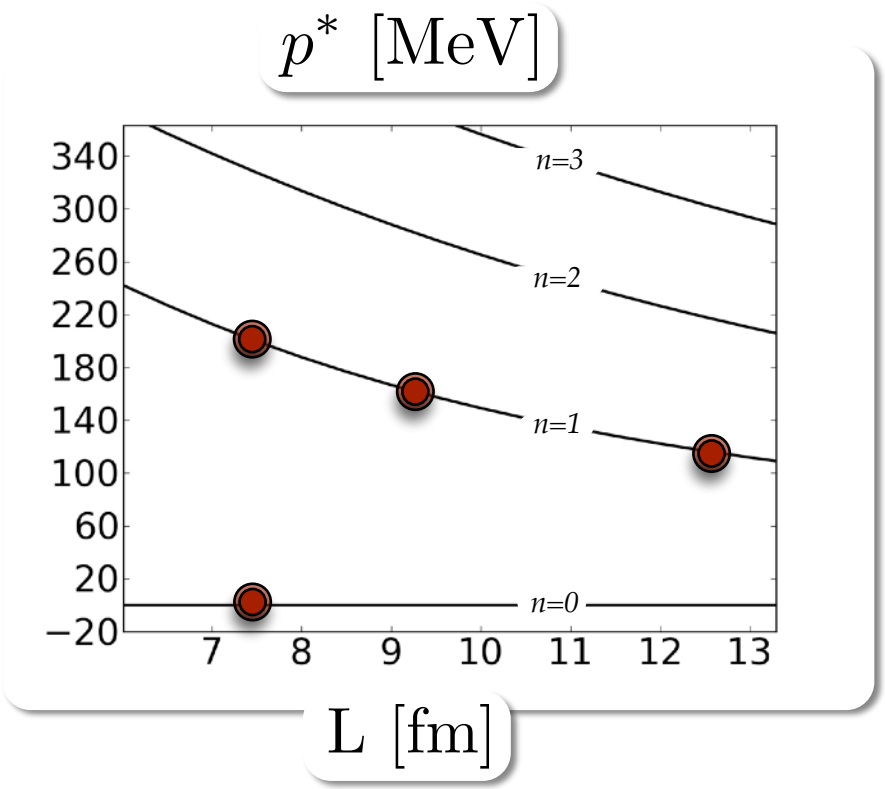

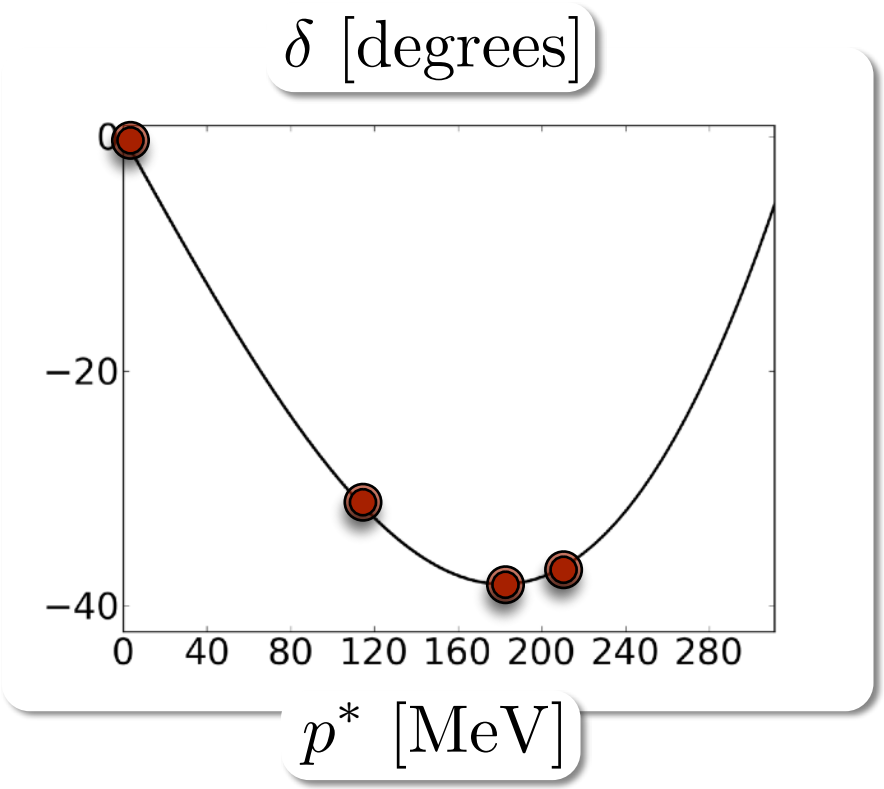

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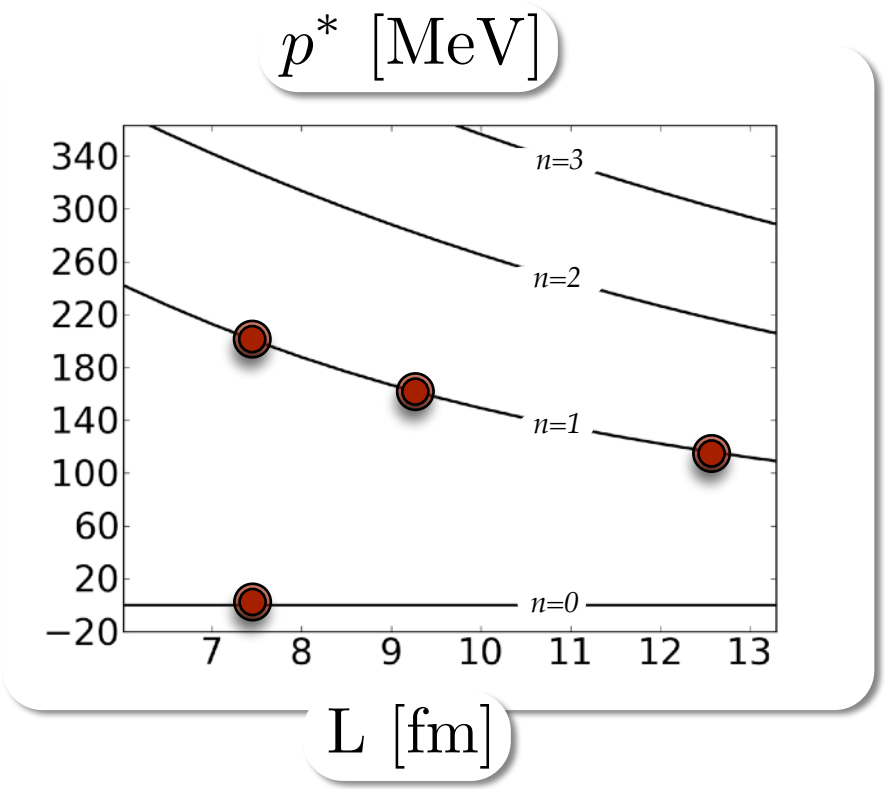
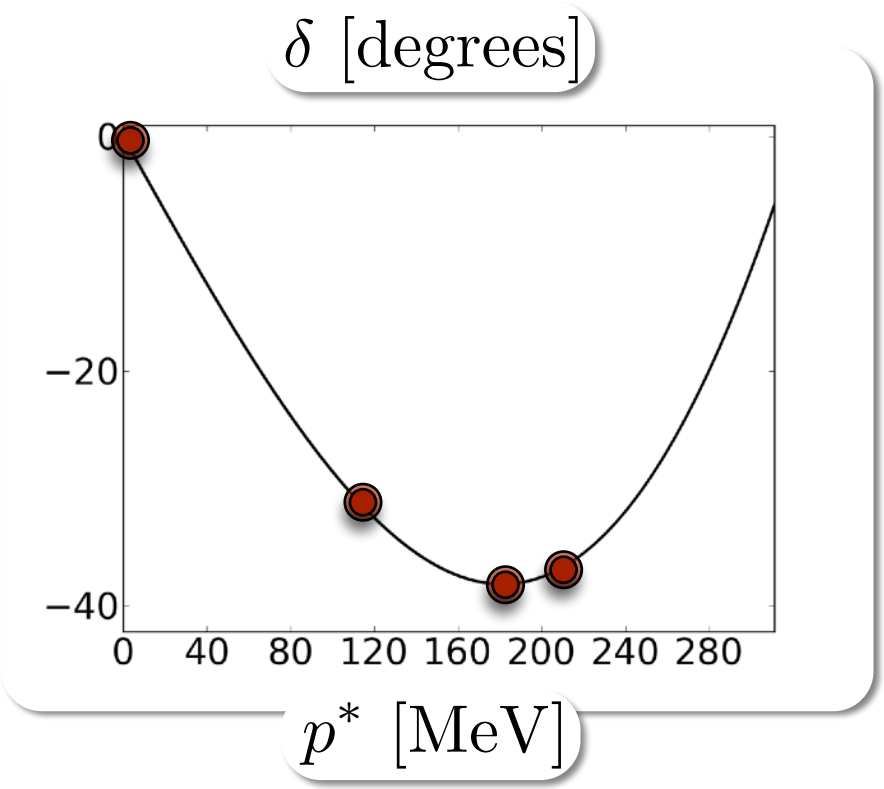




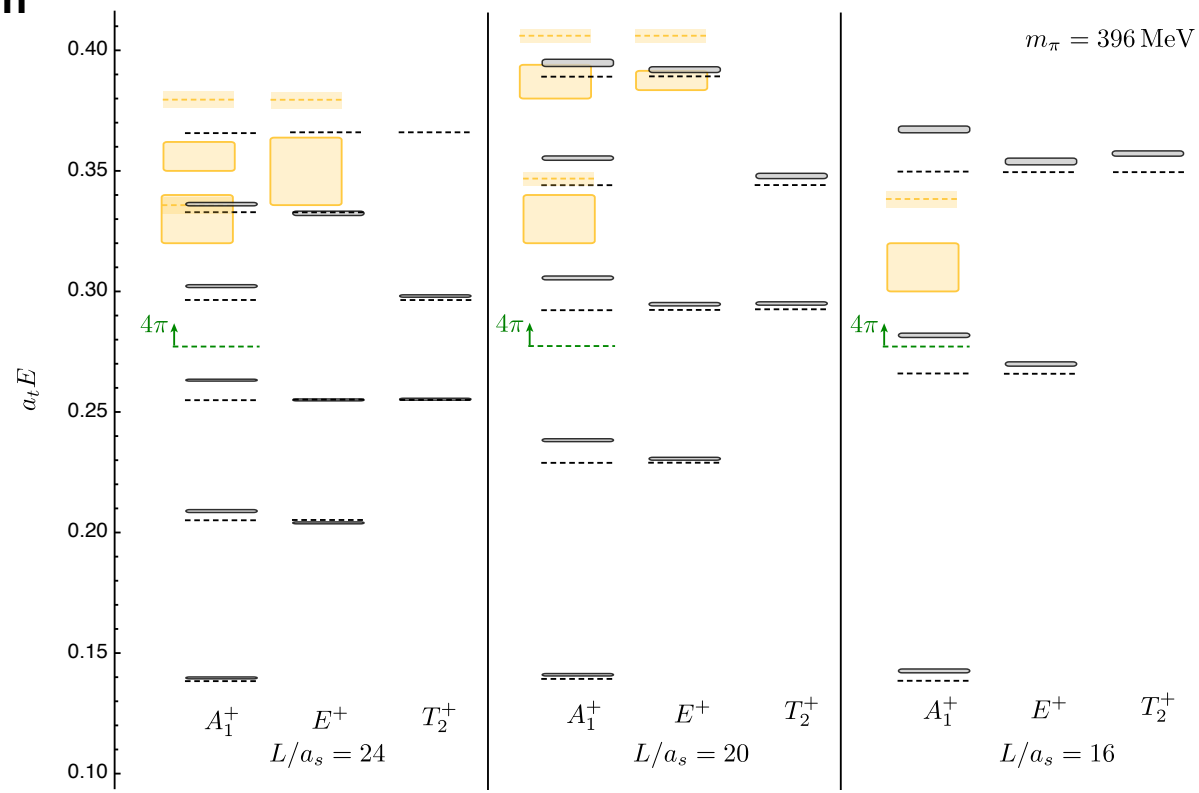
Momentum-dependent I = 2 ππ Phase Shift

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

Operator basis $$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

Total momentum zero - pion momentum ±*p*



Momentum-dependent I = 2 ππ Phase Shift

Luescher: energy levels at finite volume ↔ phase shift at corresponding *k*

$$\det\left[e^{2i\boldsymbol{\delta}(k)} - \mathbf{U}_\Gamma\left(k\frac{L}{2\pi}\right)\right] = 0$$

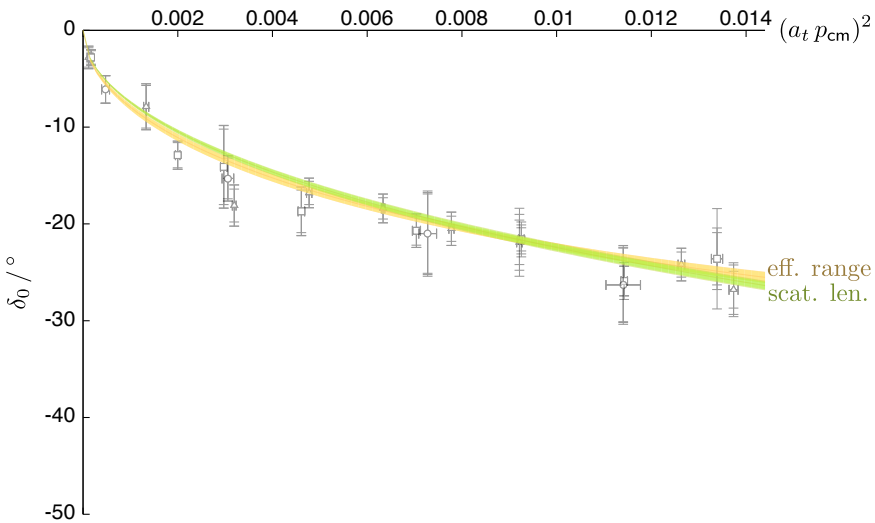
Matrix in l

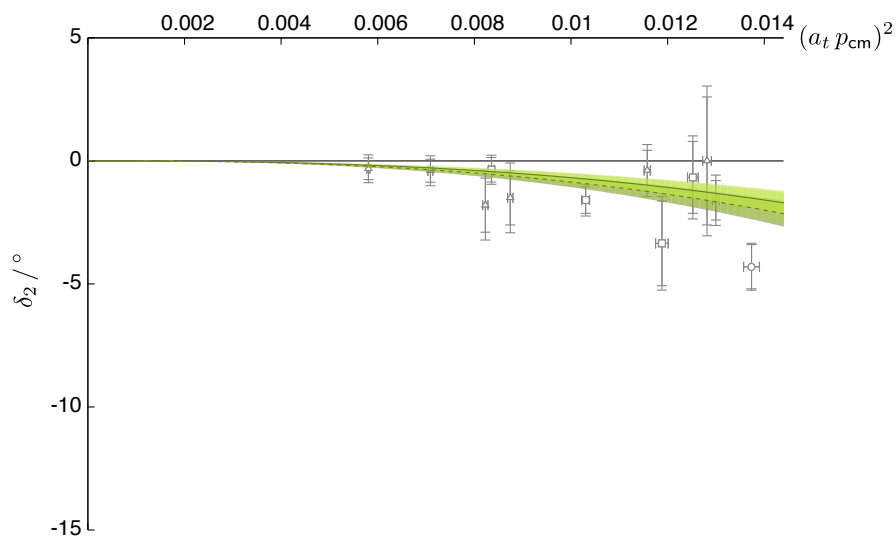
lattice irrep

Dudek *et al.*, Phys Rev D83, 071504 (2011)

Dudek, Edwards, Thomas, arXiv:1203.6041

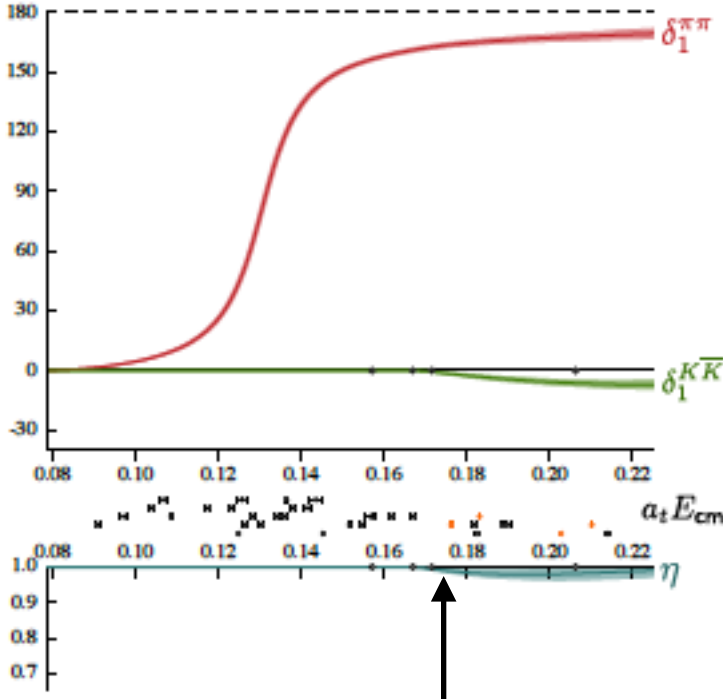
- *Moving ππ system ➙ far more momenta below inelastic threshold*
- *Optimized single-pion interpolating operators ➙ more precise determination of energies*

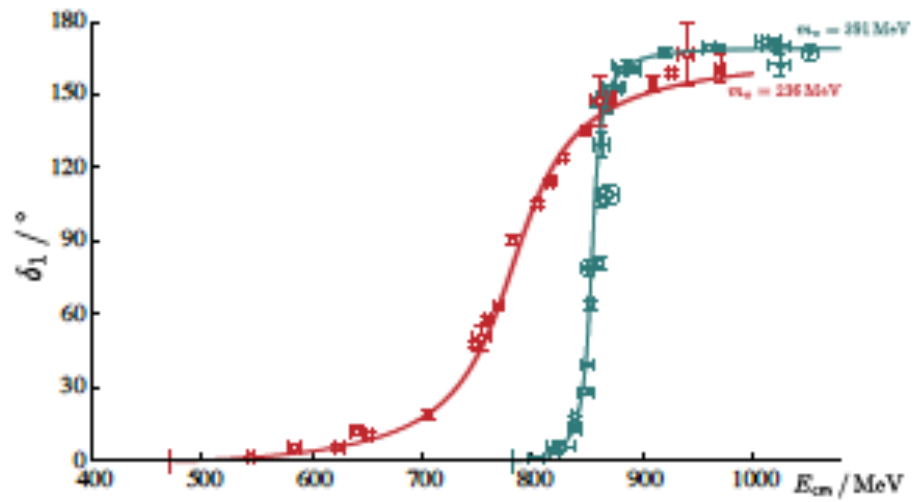




Inelastic in ππ KK channel

Wilson, Briceno, Dudek, Edwards, Thomas, arXiv:1507.02599





Decreasing Pion Mass

$$\mathcal{O}^{\Gamma,\gamma}_{\pi\pi}(|\vec{p}|) = \sum_m \mathcal{S}^{\ell,m}_{\Gamma,\gamma} \sum_{\hat{p}} Y^m_\ell(\hat{p})\, \mathcal{O}_\pi(\vec{p})\mathcal{O}_\pi(-\vec{p})$$

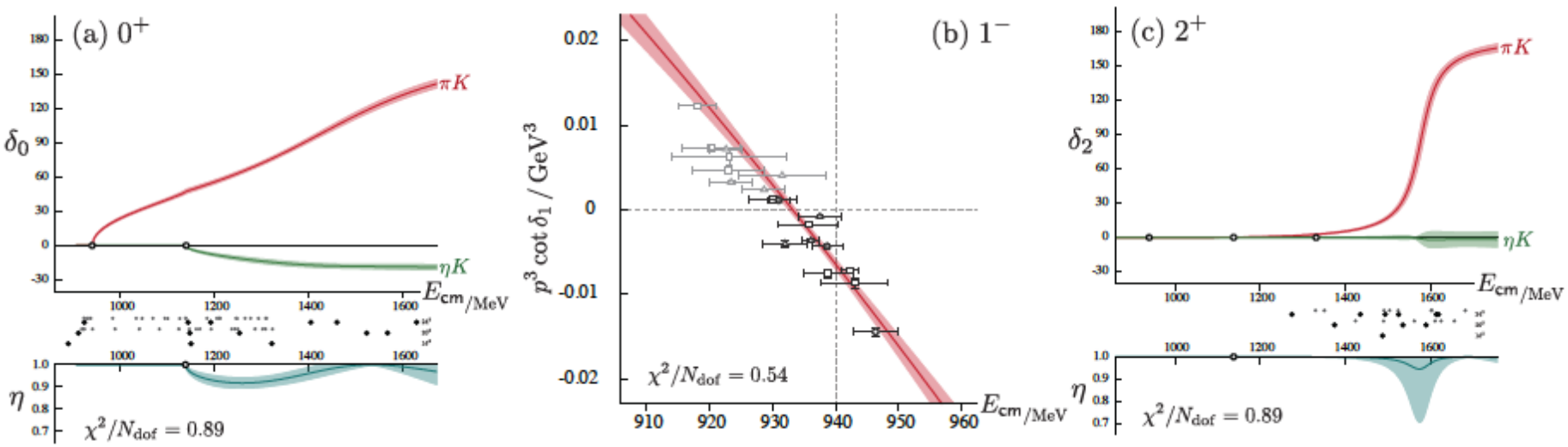
Inelastic Threshold

Inelastic

$$\det\left[\delta_{ij}\delta_{JJ'} + i\rho_i\, t_{ij}^{(J)}(E_{\text{cm}})\left(\delta_{JJ'} + i\mathcal{M}_{JJ'}^{\vec{P}\Lambda}(p_i L)\right)\right] = 0$$

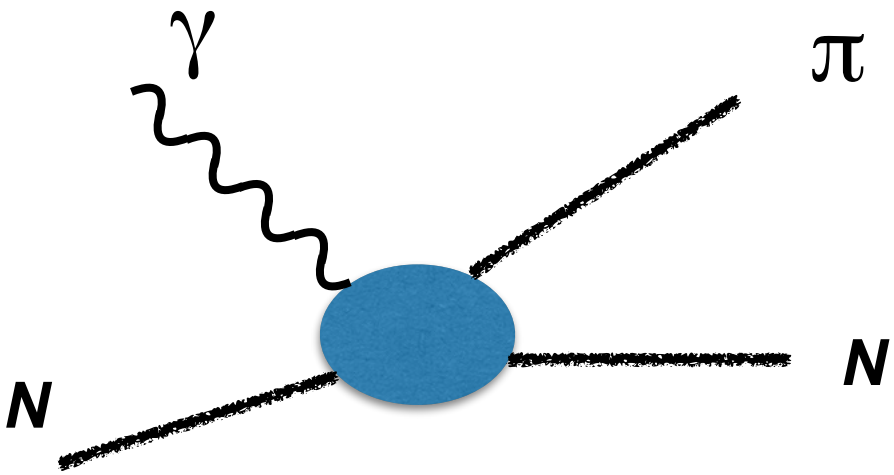
Parametrized as phase shift + inelasticity

$$t_{ii} = \frac{(\eta e^{2i\delta_i} - 1)}{2i\rho_i}, t_{ij} = \frac{\sqrt{1-\eta^2}\, e^{i(\delta_i+\delta_j)}}{2\sqrt{\rho_i\,\rho_j}}$$



Dudek, Edwards, Thomas, Wilson, PRL, PRD

Electro-Couplings



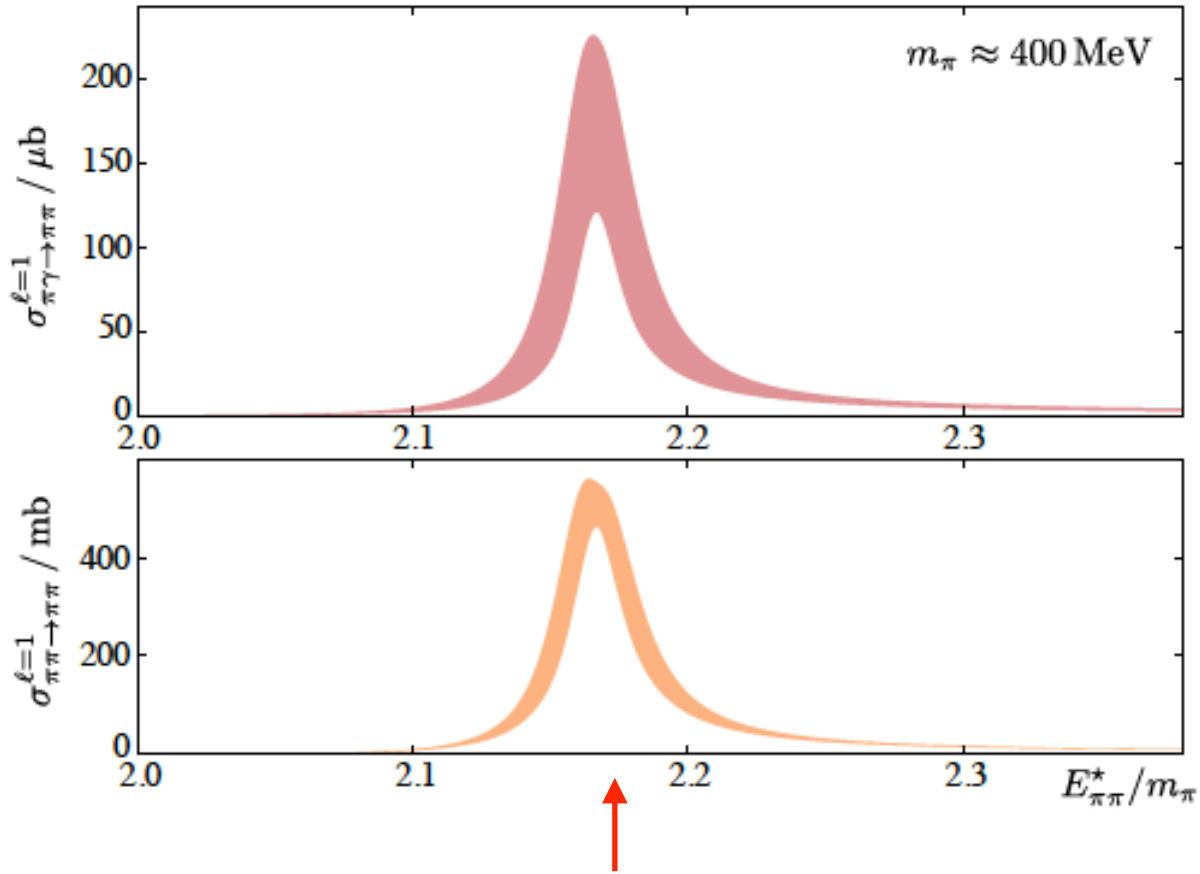
- Determination of Momentum-Dependent Amplitudes
- Formalism for 1 → 2 (3) amplitudes $\langle 1 \mid J \mid 2, 3, \ldots \rangle$

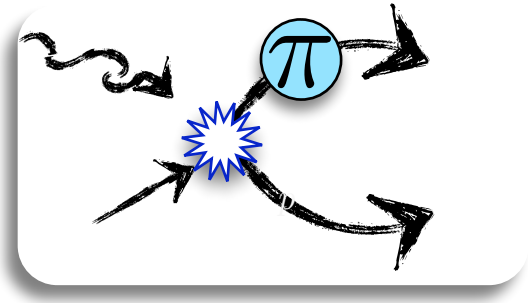
Briceno, Hansen and Walker-Loud, PRD 91, 034501 (2015)

External current: EM, Weak

$\langle 0 \mid qqq \mid 2, 3, \ldots \rangle$ Form factors at high Q^2

Transition form factor of ρ





ρ resonance

Briceno et al., Phys. Rev. D 93, 114508 (2016)

Whence baryons

Signal-to-noise ratio

$$C_\phi(t) \simeq e^{-M_\phi t}$$
$$C_{\sigma^2}(t) \simeq e^{-2m_\pi t}$$

DeGrand, Hecht, PRD46 (1992)

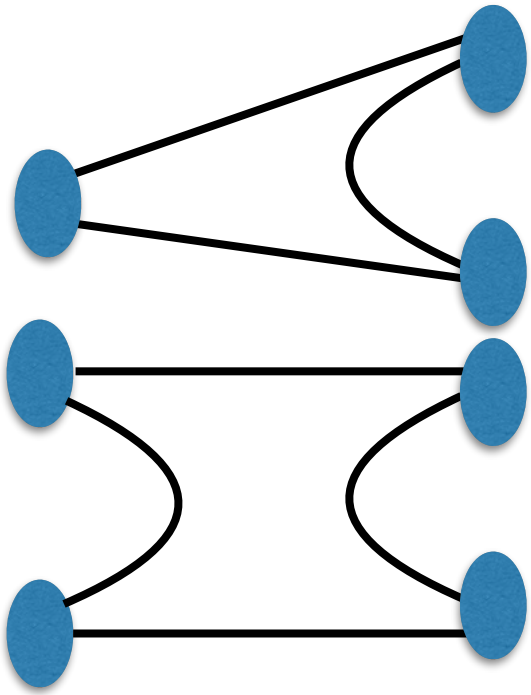
$$\text{signal-to-noise} \simeq e^{-(M_\phi - m_\pi)t}$$

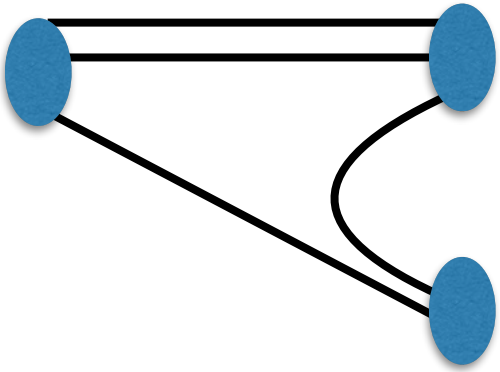
Baryons require high statistics

$$C_B(t) \simeq e^{-M_B t}$$
$$C_{\sigma^2}(t) \simeq e^{-3m_\pi t}$$

$$\text{signal-to-noise} \simeq e^{-(M_B - 3m_\pi/2)t}$$

Combinatorics and complexity



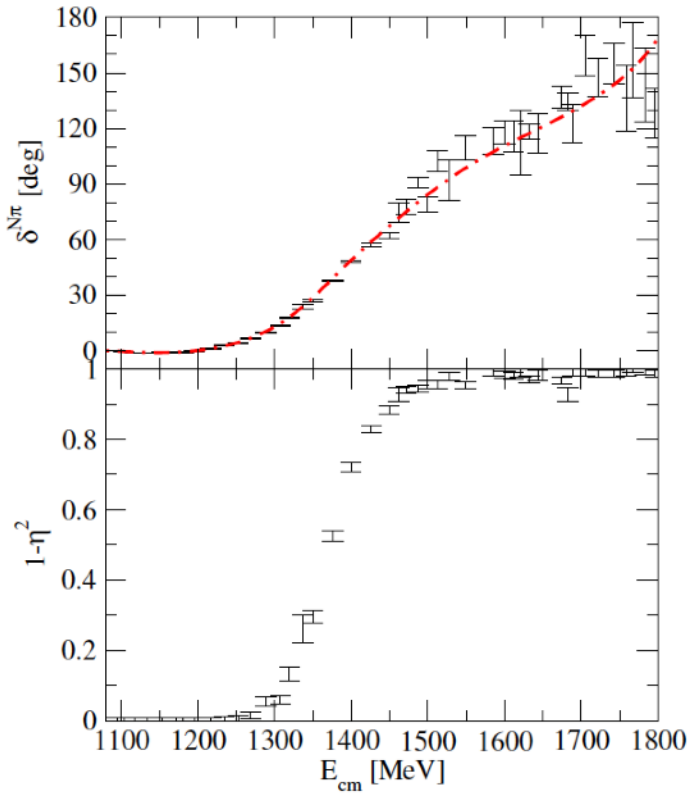


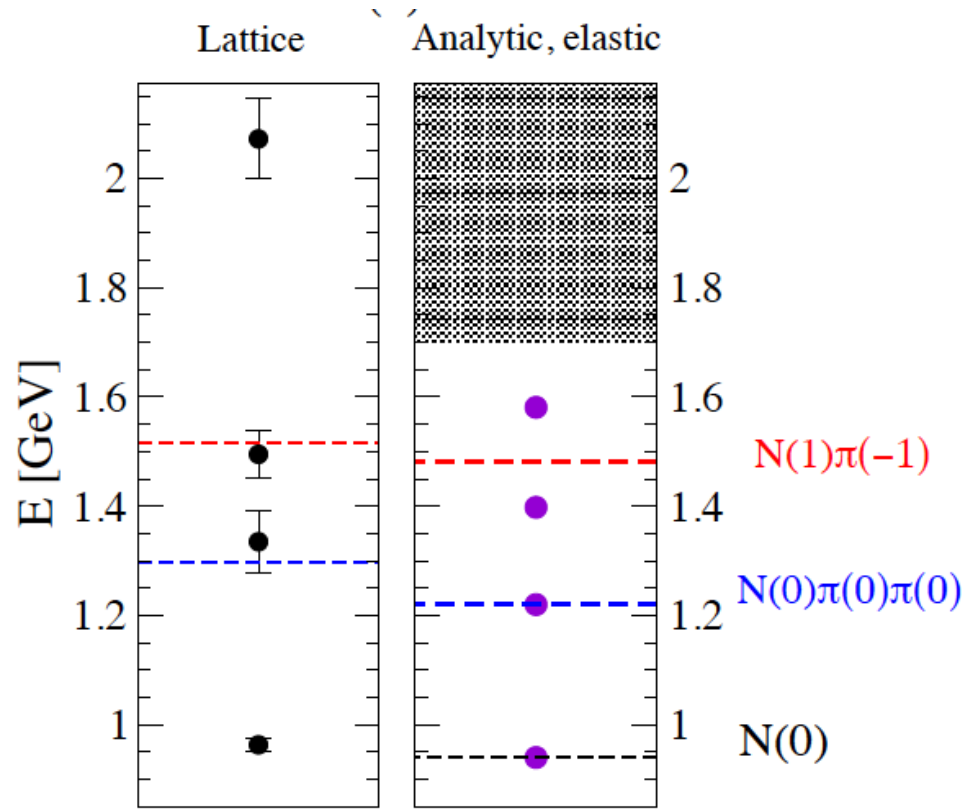
Non-integer spin

“Roper”

Observe shift in multiparticle energies

C. B. Lang, L. Leskovec, M. Padmanath, S. Prelovsek,Phys. Rev. D 95, 014510 (2017)





Strategy

Measure matrix of correlation functions: $C_{mn}(t) \equiv \sum_{\vec{x},\vec{y}} \langle N_m(\vec{x},t)\bar{N}_n(\vec{y},0)\rangle$

M. Peardon *et al.*, PRD80,054506 (2009) "Distillation"

Introduce $\tilde{\psi}(\vec{x},t) = L(\vec{x},\vec{y})\psi(\vec{y},t)$ **where L is 3D Laplacian**

Write $L \equiv (1-\kappa\nabla/n)^n = \sum_i f(\lambda_i)\xi^i \times \xi^{*i}$ **where λ_i and ξ_i are eigenvalues and eigenvectors of the Laplacian.**

Truncate* the expansion at *i* = *N_{eigen}

Insert between each quark field in our correlation function; *system at rest.*

$$C_{mn}(t) = \phi^{m,(pqr)}_{\alpha\beta\gamma)}(t)\phi^{m,(\bar{p}\bar{q}\bar{r})}_{\bar{\alpha}\bar{\beta}\bar{\gamma}}(0) \times \left[\tau^{p\bar{p}}_{\alpha\bar{\alpha}}(t,0)\tau^{q\bar{q}}_{\beta\bar{\beta}}(t,0)\tau^{r\bar{r}}_{\gamma\bar{\gamma}}(t,0) + \ldots\right]$$

Perambulators $\tau^{ij}_{\alpha\beta}(t,0) = \xi^{*i}(t)M^{-1}(t,0)_{\alpha\beta}\xi^j$ GPU, KNL

Operators $\phi^{m,(pqr)}_{\alpha\beta\gamma}$

Strategy - II

- Meson correlation functions N^3
- Baryon correlation functions N^4

Severely constrains baryon lattice sizes

Morningstar et al, Phys.Rev.D83:114505,2011

- Stochastic sampling of eigenvectors - *stochastic LaPH*
- Exploit sparsity by rotation of basis ξ → U ξ Edwards, Peardon, Winter

$$\sum_i \phi_i^*(x)\phi_i(y) = \delta(x-y), \sum_x \phi_i^*(x)\phi_j(x) = \delta_{ij}$$

- Alternative idea: simpler orthonormal basis for the smearing function L

$$\phi_i(x) = e^{-ipx}\delta_{s,s'}\delta_{c,c'}$$

Colour-wave basis

Z.Brown, K.Orginos, arXiv:1210.1953

Summary

- Formalism established for understanding resonances and electrocouplings from lattice QCD
- Methods applied and demonstrated in Meson sector
  - Computationally less demanding
  - Fewer coupled channels?
- Challenges for Baryons - computational
- Calculations at physical quark masses *not* highest priority
- Major effort to develop computational infrastructure *EXASCALE PROJECT - "contractions" (Robert Edwards)*

Mission of HadSpec