CLAS / Theory Center N Workshop*
Jefferson Lab, June 6, 2017

Connecting resonance physics to deep-inelastic phenomena: *studies in quark-hadron duality*

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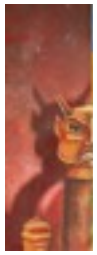
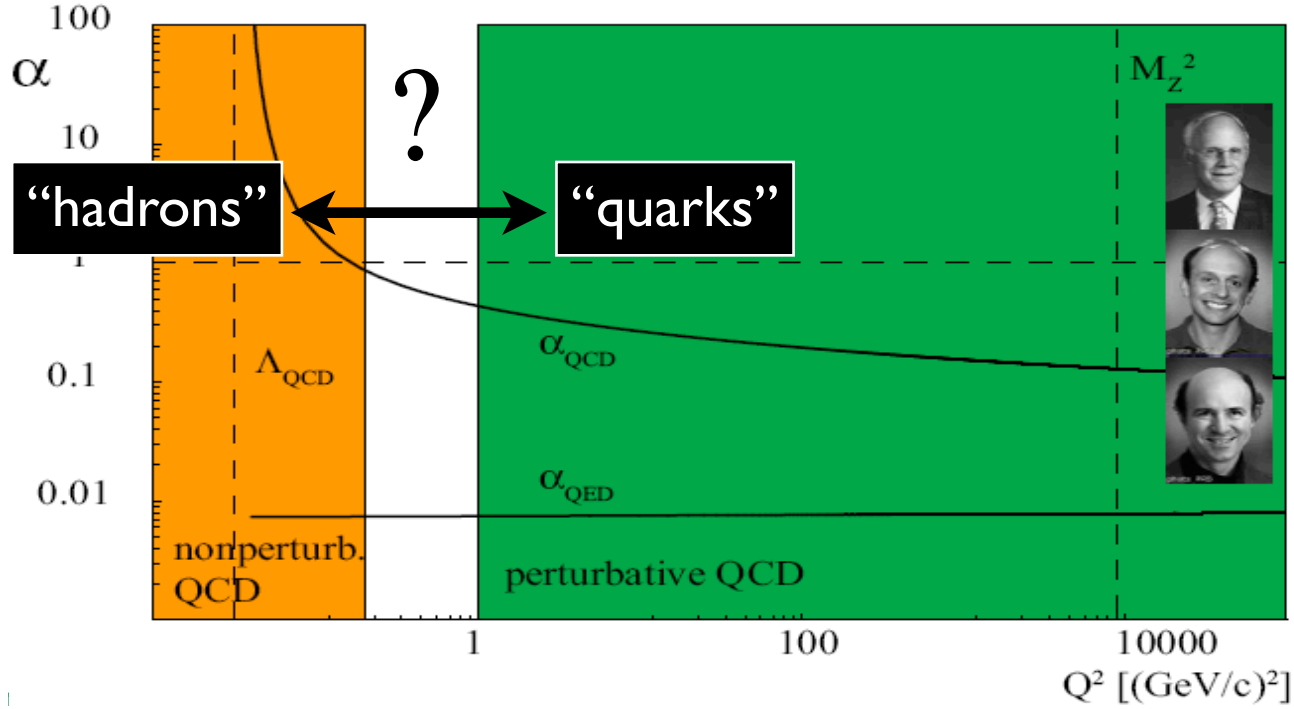
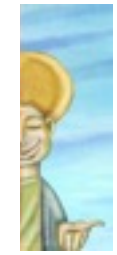
Jefferson Lab







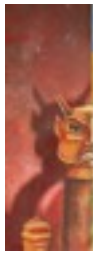
low energy *long distance*

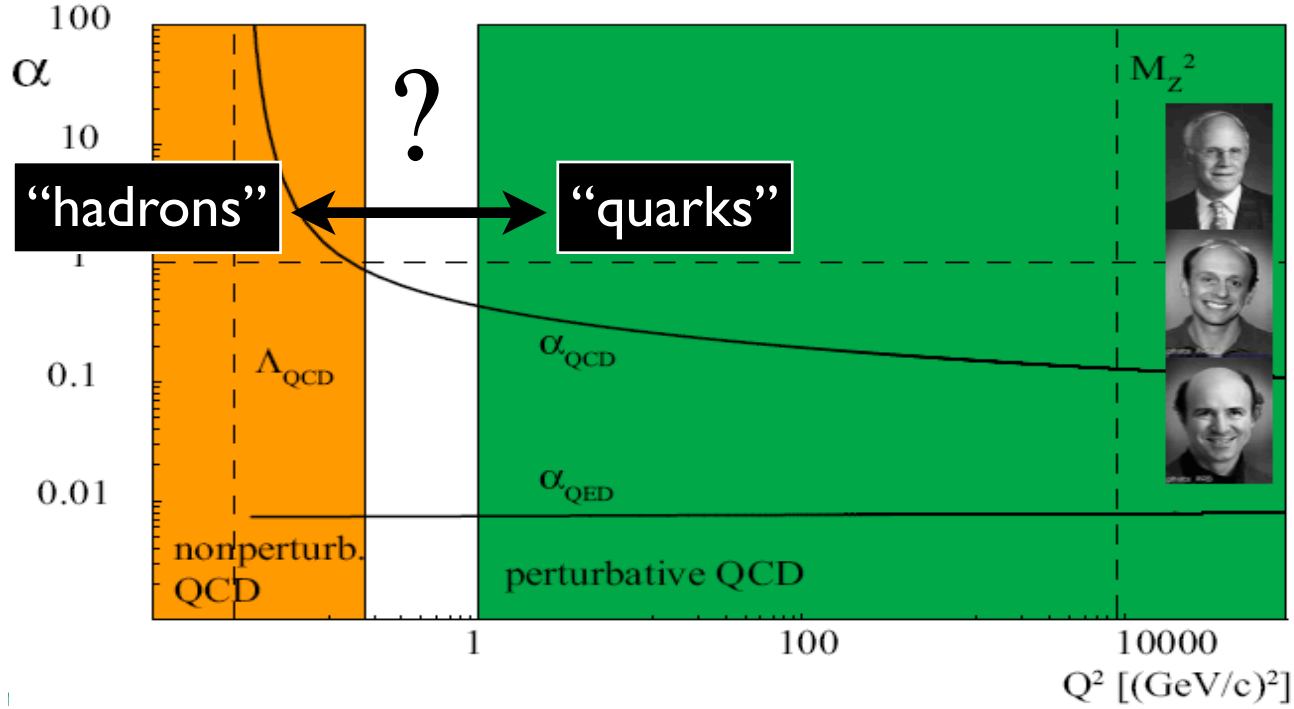
high energy *short distance*

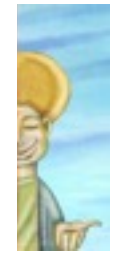
- Duality hypothesis: complementarity between *quark* and *hadron* descriptions of observables

$$\sum_{hadrons} = \sum_{quarks}$$

→ can use either set of *complete* basis states to describe physical phenomena







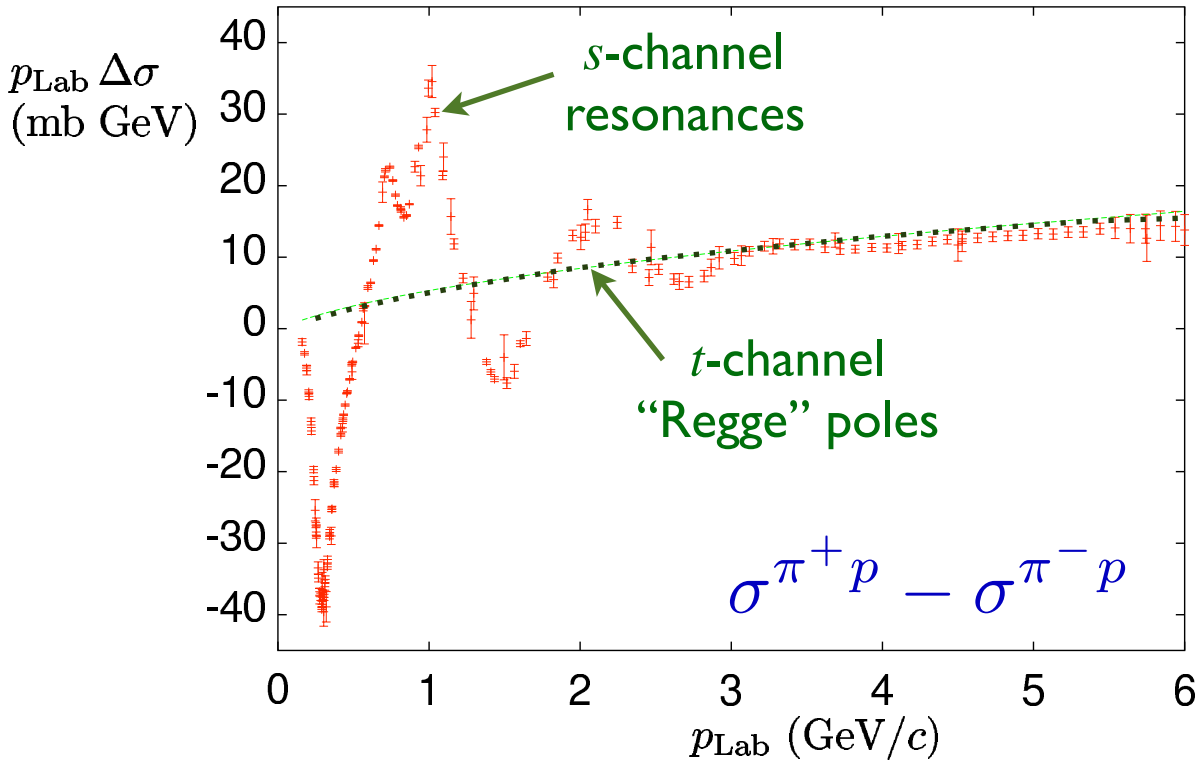
low energy *long distance*

high energy *short distance*

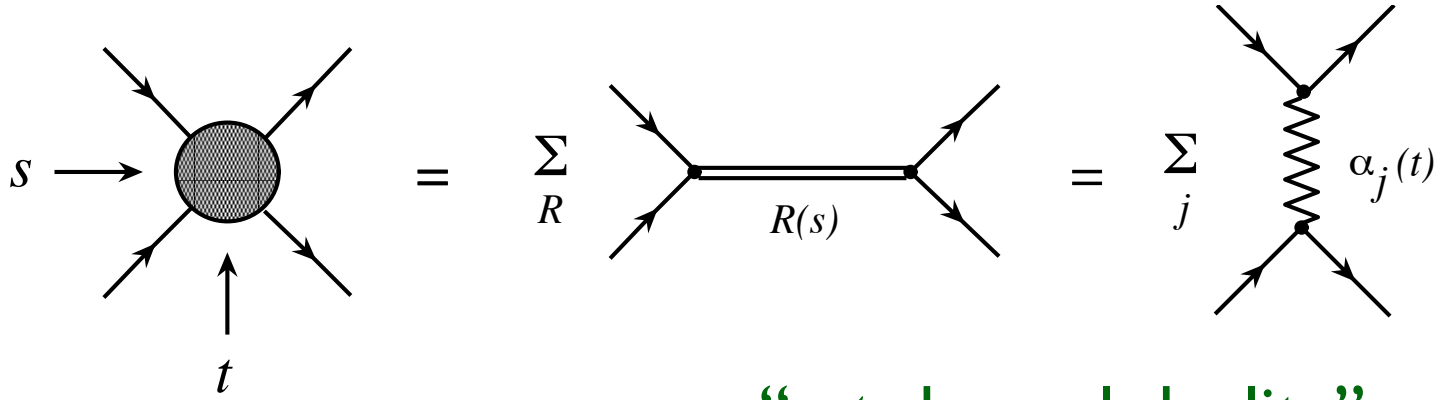
- In practice, at finite energy typically have access only to *limited* set of basis states

- Question is not *why* duality exists, but *how* it arises where it exists, and how can we make use of it?

Duality in hadron-hadron scattering



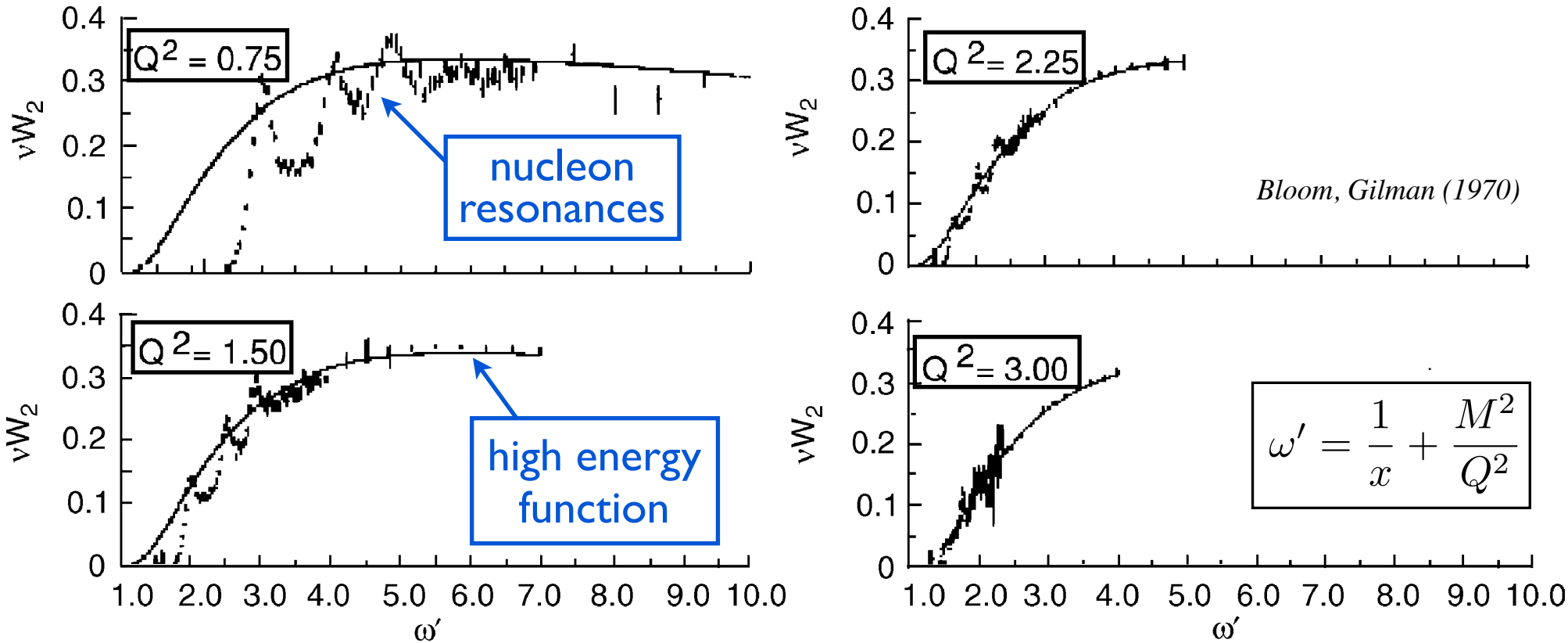
Igi (1962)
Dolen, Horn, Schmidt (1968)



"s-t channel duality"

Duality in electron-nucleon scattering

"Bloom-Gilman duality"

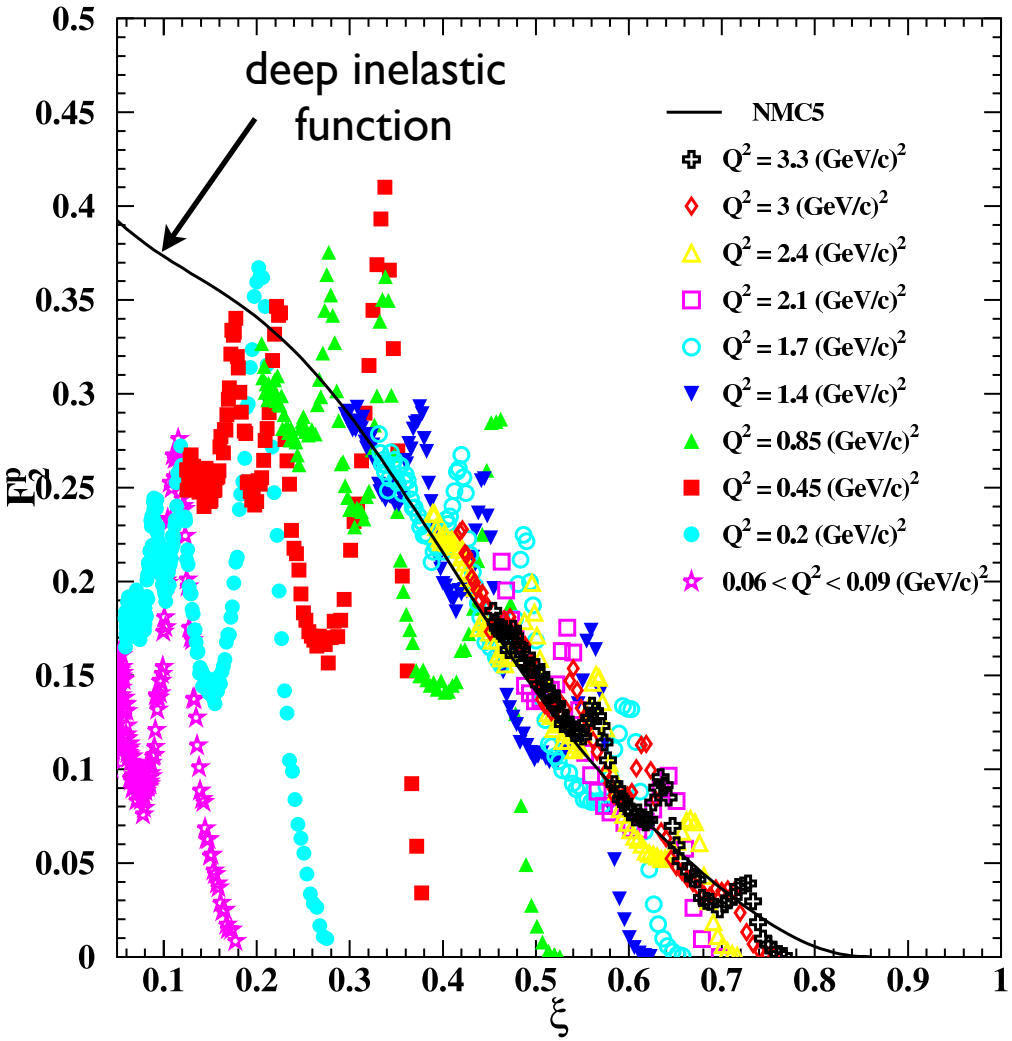


$$\omega' = \frac{1}{x} + \frac{M^2}{Q^2}$$

"hadrons" $$\frac{2M}{Q^2}\int_0^{\nu_m} d\nu\ \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega'\ \nu W_2(\omega')$$ "quarks"

"finite-energy sum rules"

Duality in electron-nucleon scattering



$$\xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}$$

average over resonances

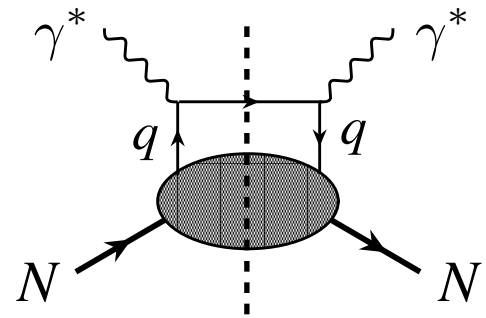
(strongly Q^2 dependent)

$\approx$ Q^2 independent scaling function

Duality in electron-nucleon scattering

- In *deep-inelastic* region ($W \gtrsim 2$ GeV, $Q^2 \gtrsim 1$ GeV^2) structure function given by parton distributions

$$F_2(x, Q^2) \stackrel{\text{LO}}{=} x \sum_q e_q^2 \, q(x, Q^2)$$



- In *resonance* region ($W \lesssim 2$ GeV), or at low Q^2 ($Q^2 \lesssim 1$ GeV^2) can no longer resolve individual quark structure

- Resonance and DIS regions intimately connected

  $\longrightarrow$ resonances an *integral* part of scaling structure function

  e.g. in large-N_c limit, spectrum of zero-width resonances is "maximally dual" to quark-level (smooth) structure function

Scaling functions from resonances

- Earliest attempts predate QCD

$\longrightarrow$ $e.g.$ harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = $1/2$, ..., n+$1/2$
(n even: $I = 1/2$, n odd: $I = 3/2$)

Domokos et al. (1971)

$\longrightarrow$ at large Q^2 magnetic coupling dominates

$$G_n(Q^2) = \frac{\mu_n}{(1 + Q^2 r^2/M_n^2)^2} \qquad r^2 \approx 1.41$$

$\longrightarrow$ in Bjorken limit, $\sum_n \longrightarrow \int dz\,, \quad z \equiv M_n^2/Q^2$

$$F_2 \sim (\omega' - 1)^{1/2}(\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2 z^2}$$

$\longrightarrow$ scaling function of $\omega' = \omega + M^2/Q^2 \qquad (\omega = 1/x)$

Scaling functions from resonances

- Earliest attempts predate QCD

→ *e.g.* harmonic oscillator spectrum $M_n^2 = (n+1)\Lambda^2$
including states with spin = $1/2, \ldots, n+1/2$
(n even: $I = 1/2$, n odd: $I = 3/2$)

Domokos et al. (1971)

→ in $\Gamma_n \to 0$ limit

$$F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2)\frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4}$$

cf. Drell-Yan-West relation

$$G(Q^2) \sim \left(\frac{1}{Q^2}\right)^m \iff F_2(x) \sim (1-x)^{2m-1}$$

→ similar behavior found in many models

Einhorn (1976) ('t Hooft model)
Greenberg (1993) (NR scalar quarks in HO potential)
Pace, Salme, Lev (1995) (relativistic HO with spin)
Isgur et al. (2001) (transition to scaling)
....

Scaling functions from resonances

- Phenomenological analyses at finite Q^2

$\longrightarrow$ additional constraints from threshold behavior at $\mathbf{q} \to 0$ and asymptotic behavior at $Q^2 \to \infty$

$$\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M\nu \; \left[|G_+^R|^2 + 2|G_0^R|^2 + |G_-^R|^2\right] \; \delta(W^2 - M_R^2)$$

Davidovsky, Struminsky (2003)

$\longrightarrow$ 21 isospin-$1/2$ & $3/2$ resonances (with mass < 2 GeV)

$$\left|G_\pm^R(Q^2)\right|^2 = \left|G_\pm^R(0)\right|^2 \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_1} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_\pm} \qquad m_{+,0,-} = 3, 4, 5$$

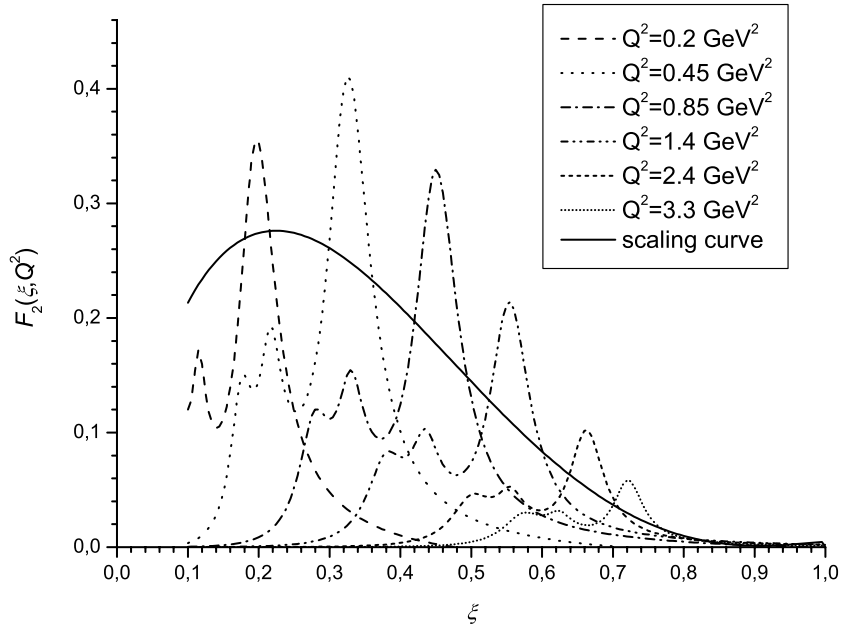
$$\left|G_0^R(Q^2)\right|^2 = C^2 \left(\frac{Q^2}{Q^2 + \Lambda''^2}\right)^{2a} \frac{q_0^2}{|\vec{q}|^2} \left(\frac{|\vec{q}|}{|\vec{q}|_0} \frac{\Lambda'^2}{Q^2 + \Lambda'^2}\right)^{\gamma_2} \left(\frac{\Lambda^2}{Q^2 + \Lambda^2}\right)^{m_0}$$

$\longrightarrow$ in $x \to 1$ limit,

$$F_2(x) \sim (1 - x)^{m_+}$$

Scaling functions from resonances

- Phenomenological analyses at finite Q^2

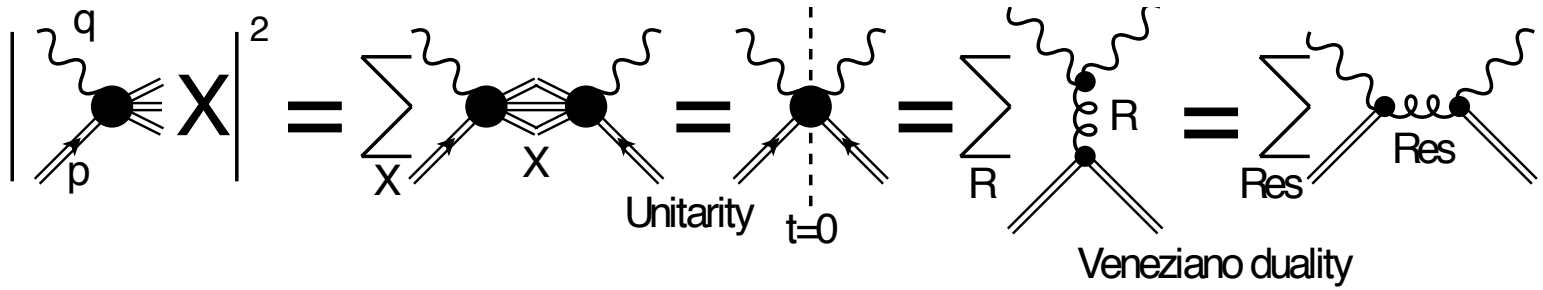


$\longrightarrow$ valence-like structure of dual function suggests "two-component duality":

- valence (Reggeon exchange) dual to resonances $F_2^{(\mathrm{val})} \sim x^{0.5}$
- sea (Pomeron exchange) dual to background $F_2^{(\mathrm{sea})} \sim x^{-0.08}$

Scaling functions from resonances

- Explicit realization of Veneziano & Bloom-Gilman duality



$$V(s,t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

$\rightarrow \; s^{\alpha(t)}$ high s, low $|t|$

$\longrightarrow$ Veneziano model not unitary, has no imaginary parts

$\longrightarrow$ generalization of narrow-resonance approximation, with nonlinear, complex Regge trajectories

$$D(s,t) = \int_0^1 dz \left(\frac{z}{g}\right)^{-\alpha_s(s(1-z))-1} \left(\frac{1-z}{g}\right)^{-\alpha_t(tz)-1}$$

"dual amplitude with Mandelstam analyticity" (DAMA) model

Jenkovszky et al.

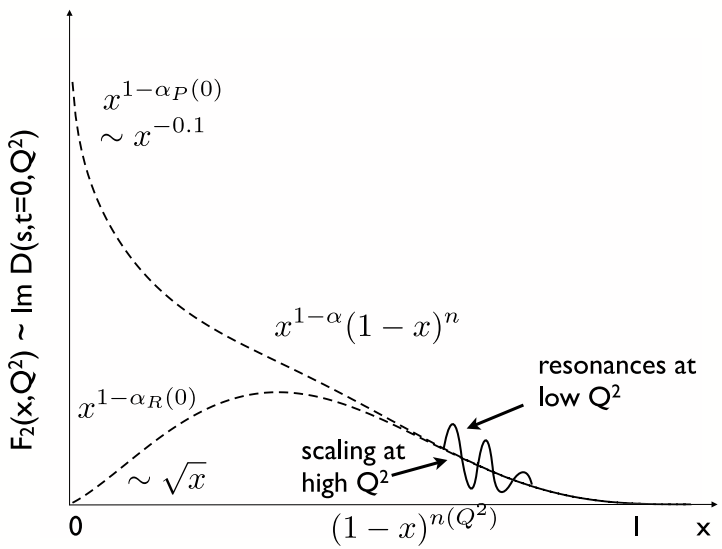
Scaling functions from resonances

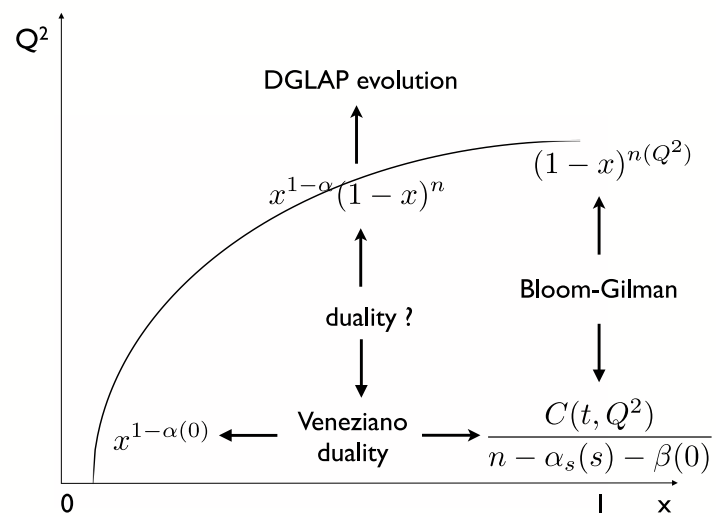
- Explicit realization of Veneziano & Bloom-Gilman duality

$\longrightarrow$ for large x and Q^2, have power-law behavior

$$F_2 \sim (1-x)^{2\alpha_t(0)\ln 2g/\ln g}$$

where parameter g can be Q^2 dependent





Jenkovszky, Magas, Londergan, Szczepaniak (2012)

Duality and QCD

- Operator product expansion

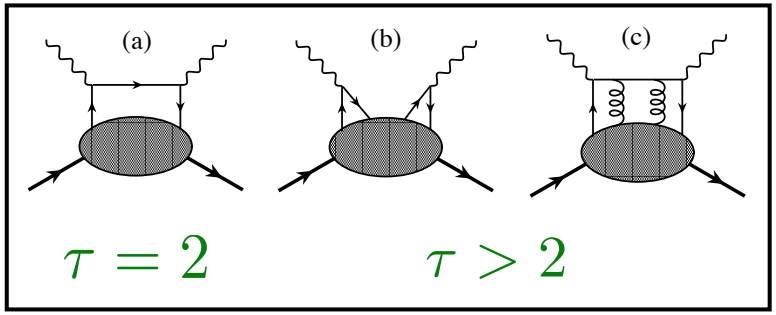
$\longrightarrow$ expand *moments* of structure functions in powers of $1/Q^2$

$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$

$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

matrix elements of operators with specific "twist" τ

$$\tau = \text{dimension} - \text{spin}$$



Duality and QCD

- Operator product expansion

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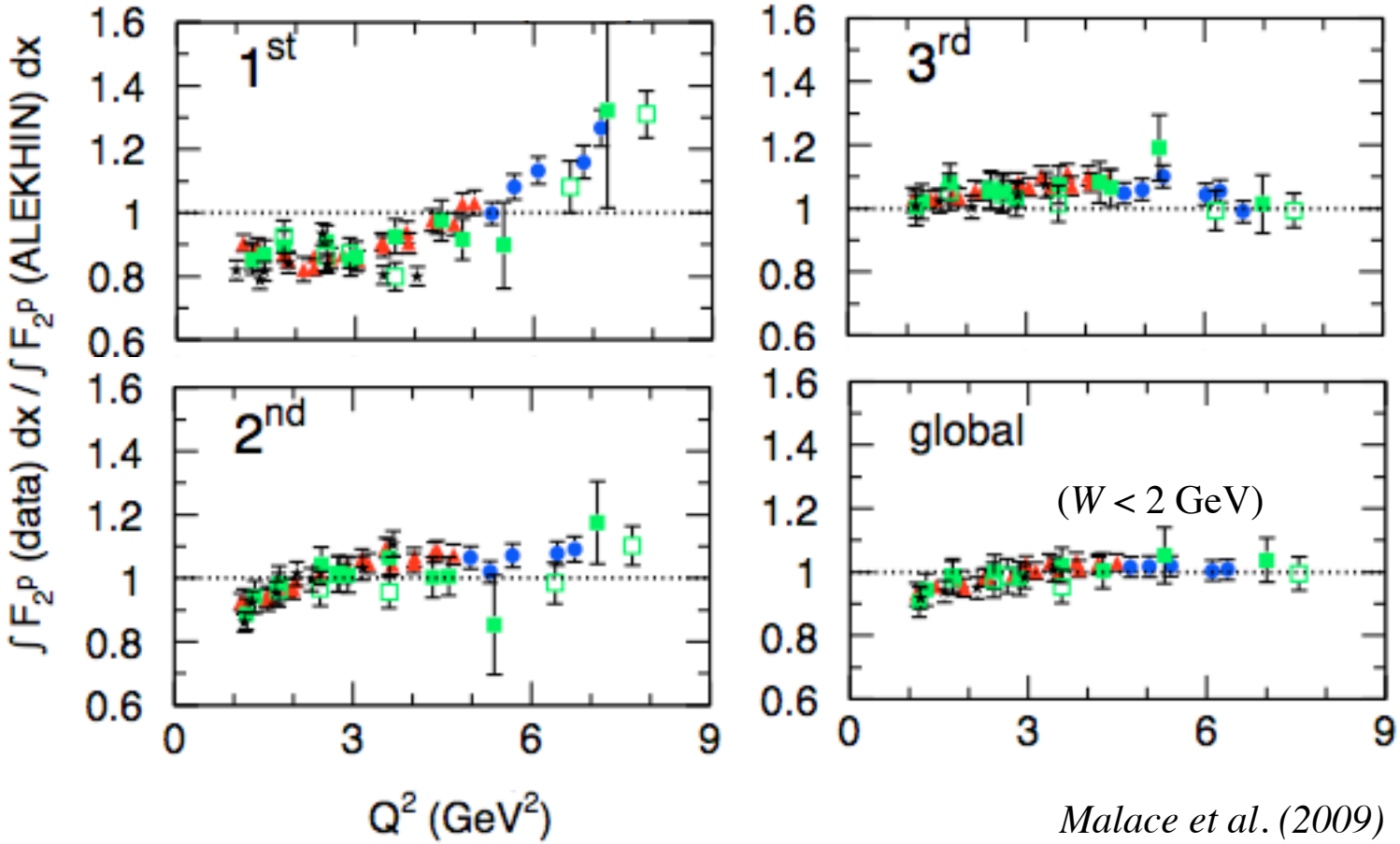
$$M_n(Q^2) = \int_0^1 dx\; x^{n-2}\; F_2(x, Q^2)$$
$$= A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots$$

- If moment $\approx$ independent of Q^2

  $\longrightarrow$ "higher twist" terms $A_n^{(\tau>2)}$ small

- Duality $\longleftrightarrow$ suppression of higher twists

Truncated moments of F_2^p in resonance region



Malace et al. (2009)

$\longrightarrow$ higher twists $< 10\text{–}15\%$ for $Q^2 > 1\ \mathrm{GeV}^2$

Resonances & twists

- Total "higher twist" is *small* at scales $Q^2 \sim \mathcal{O}(1\ \mathrm{GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)

  $\longrightarrow$ nontrivial interference between resonances

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- Can we understand this dynamically, at quark level?

  $\longrightarrow$ is duality an accident?

- Can we use resonance region data to learn about *leading twist* structure functions (and *vice versa*)?

  $\longrightarrow$ expanded data set has potentially significant implications for global quark distribution studies

- Consider simple quark model with spin-flavor symmetric wave function

<u>low energy</u>

$\longrightarrow$ *coherent* scattering from quarks $\quad d\sigma \sim \left(\sum_i e_i \right)^2$

<u>high energy</u>

$\longrightarrow$ *incoherent* scattering from quarks $\quad d\sigma \sim \sum_i e_i^2$

- For duality to work, these must be equal

$\longrightarrow$ how can *square of a sum* become *sum of squares*?

Dynamical cancellations

→ *e.g.* for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left|G_{0,n}(\mathbf{q}^2)\right|^2 \delta(E_n - E_0 - \nu)$$

→ charge operator $\Sigma_i \; e_i \exp(i\mathbf{q} \cdot \mathbf{r}_i)$ excites
even partial waves with strength $\propto (e_1 + e_2)^2$
odd partial waves with strength $\propto (e_1 - e_2)^2$

→ resulting structure function

$$F(\nu, \mathbf{q}^2) \sim \sum_n \left\{(e_1 + e_2)^2 \; G^2_{0,2n} + (e_1 - e_2)^2 \; G^2_{0,2n+1}\right\}$$

→ if states degenerate, *cross terms* $(\sim e_1 e_2)$ *cancel* when averaged over nearby *even and odd parity* states

Close, Isgur (2001)

Dynamical cancellations

→ duality is realized by summing over at least one complete set of *even* and *odd* parity resonances

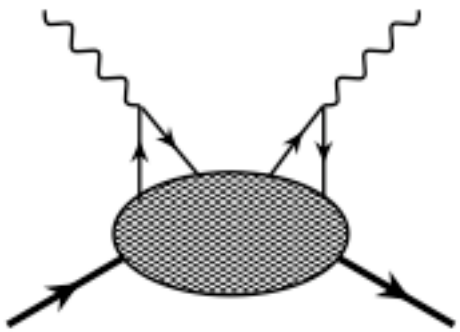
→ in NR Quark Model, even & odd parity states generalize to **56** (L=0) and **70** (L=1) multiplets of spin-flavor SU(6)

representation	$^2\mathbf{8}[\mathbf{56}^+]$	$^4\mathbf{10}[\mathbf{56}^+]$	$^2\mathbf{8}[\mathbf{70}^-]$	$^4\mathbf{8}[\mathbf{70}^-]$	$^2\mathbf{10}[\mathbf{70}^-]$	Total
F_1^p	$9\rho^2$	$8\lambda^2$	$9\rho^2$	0	λ^2	$18\rho^2+9\lambda^2$
F_1^n	$(3\rho+\lambda)^2/4$	$8\lambda^2$	$(3\rho-\lambda)^2/4$	$4\lambda^2$	λ^2	$(9\rho^2+27\lambda^2)/2$

λ (ρ) = (anti) symmetric component of ground state wave function

Close, WM (2003, 2009)

Accidental cancellations of charges?



cat's ears diagram (4-fermion higher twist $\sim 1/Q^2$)

$$\propto \sum_{i \neq j} e_i \, e_j \sim \Big(\sum_i e_i\Big)^2 - \sum_i e_i^2$$

coherent *incoherent*

proton $\quad \mathrm{HT} \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0\,!$

neutron $\quad \mathrm{HT} \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0$

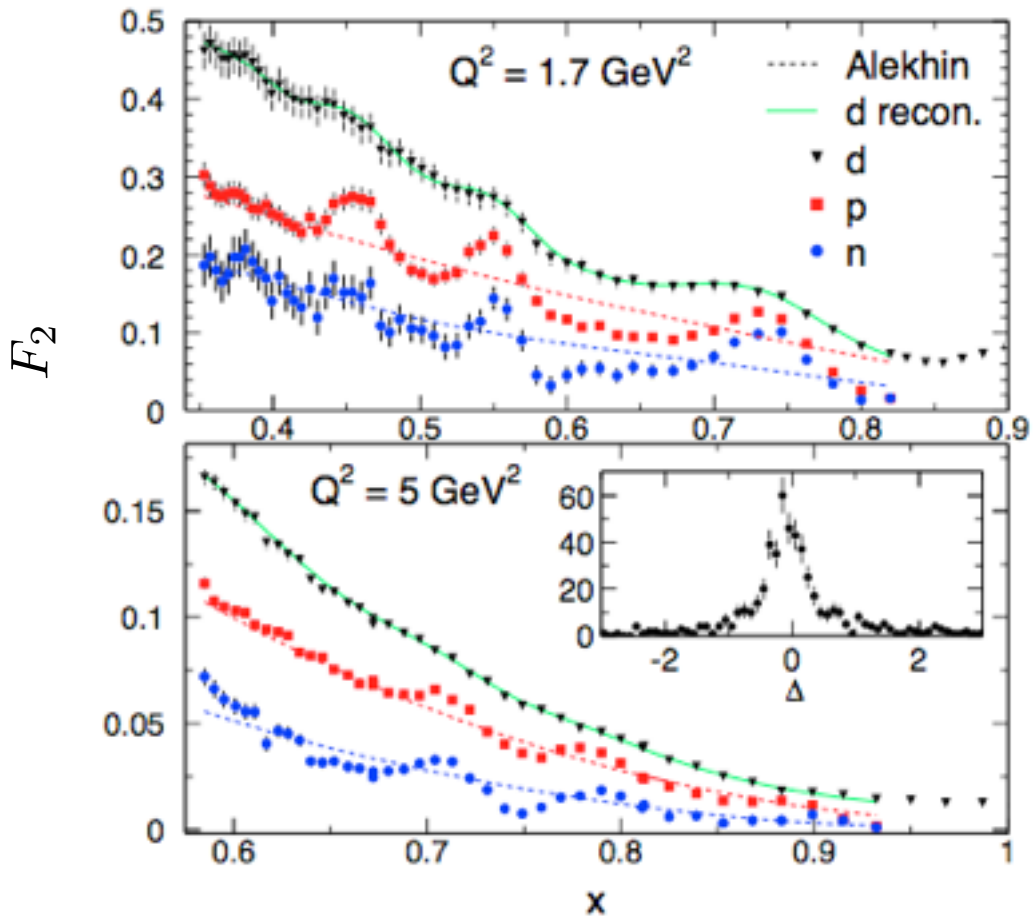
$\longrightarrow$ duality in proton a *coincidence*!

$\longrightarrow$ should *not* hold for neutron !!

Brodsky (2000)

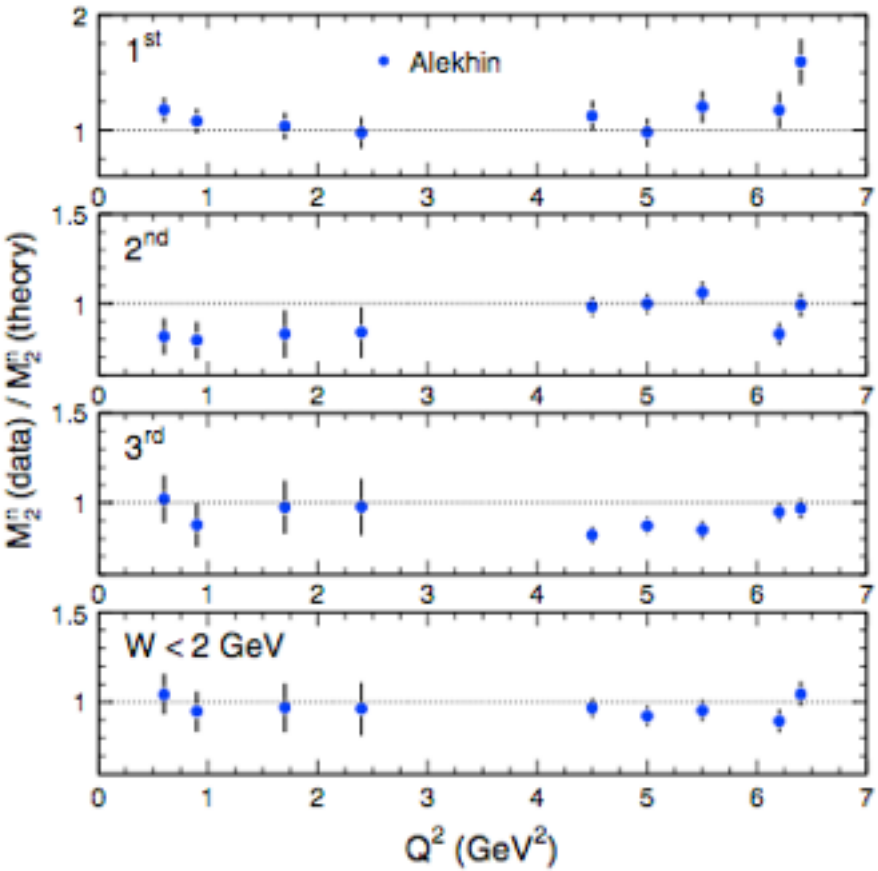
Neutron: the smoking gun

- Duality in *neutron* more difficult to test because of absence of free neutron targets
- New extraction method (using iterative procedure for solving integral convolution equations) allowed first determination of F_2^n in resonance region & test of neutron duality



Malace et al. (2010)

Neutron: the smoking gun



→ "theory": global QCD fit to $W > 2$ GeV data

→ *locally*, violations of duality in resonance regions $< 15-20\%$ (largest in Δ region)

→ *globally*, violations $< 10\%$

→ duality is not accidental, but a general feature of resonance–scaling transition!

→ use resonance region data to learn about *leading twist* structure functions?

CTEQ-JLab (CJ) global PDF analysis

- Global QCD analysis of high-energy scattering data, including large-x, low-Q^2 region

- Systematically study effects of Q^2 & W cuts

cut0: $Q^2 > 4\ \mathrm{GeV}^2, \quad W^2 > 12.25\ \mathrm{GeV}^2$

cut1: $Q^2 > 3\ \mathrm{GeV}^2, \quad W^2 > 8\ \mathrm{GeV}^2$

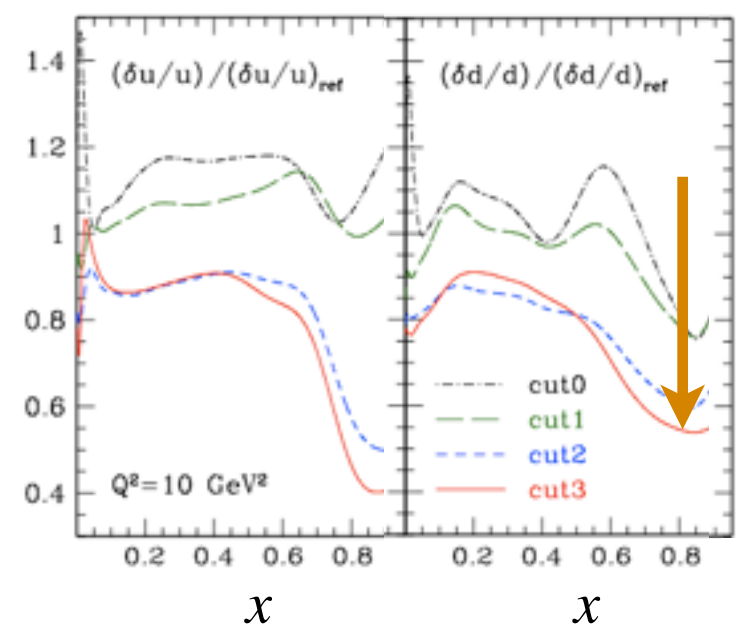
cut2: $Q^2 > 2\ \mathrm{GeV}^2, \quad W^2 > 4\ \mathrm{GeV}^2$

cut3: $Q^2 > m_c^2\ , \quad W^2 > 3\ \mathrm{GeV}^2$

factor 2 increase in DIS data from cut0 → cut3

→ larger database with weaker cuts gives significantly reduced errors, especially at large x

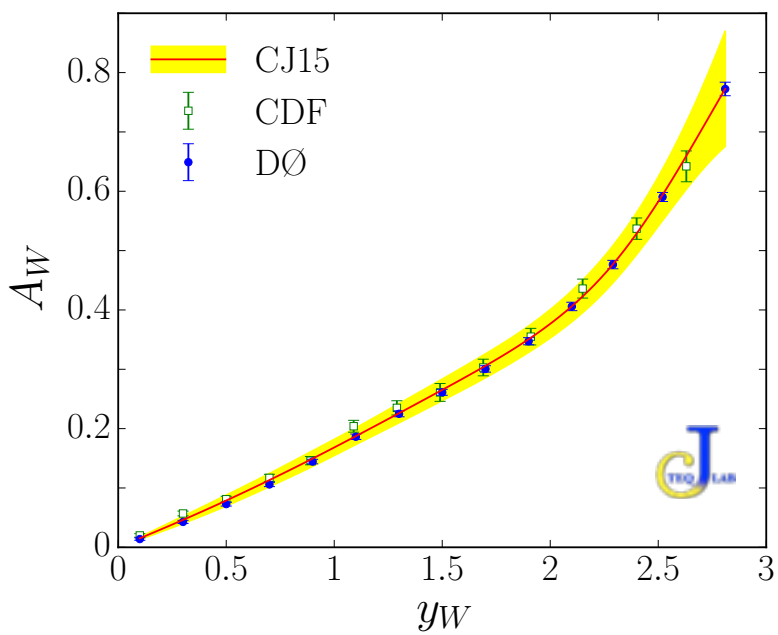
→ up to ~ 40–60% error reduction when cuts extended into near-resonance region



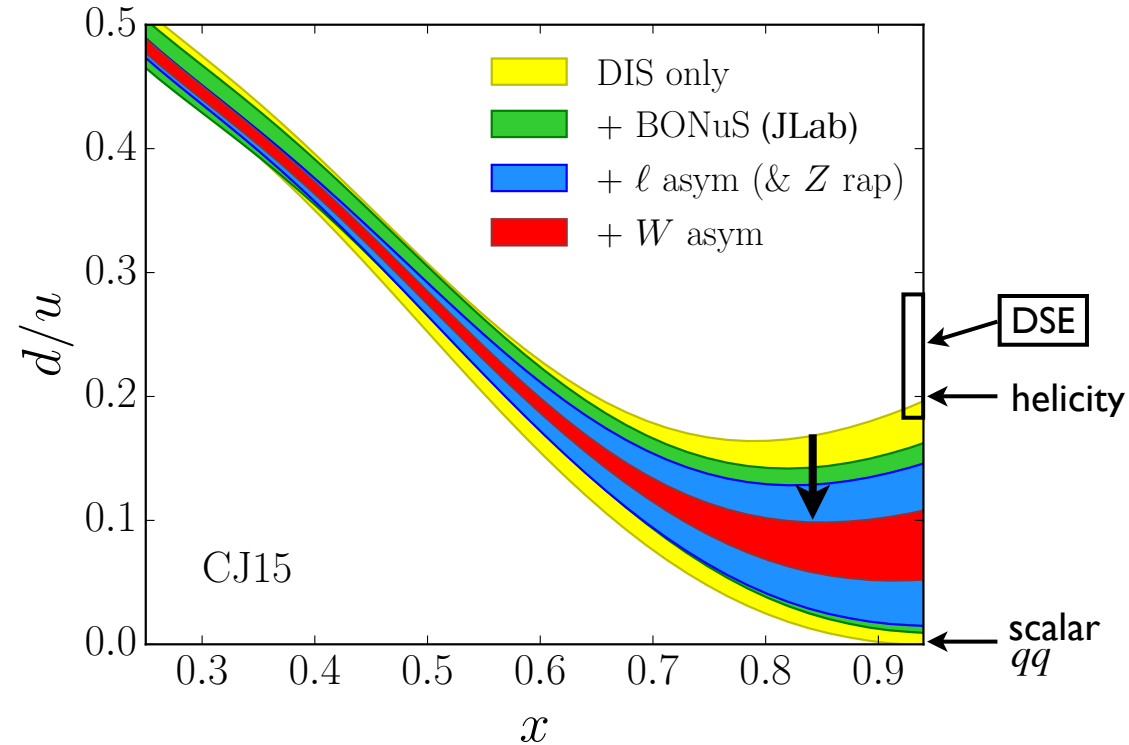
CTEQ-JLab (CJ) global PDF analysis

- Valence d/u ratio at high x

→ significant reduction of PDF errors with new JLab tagged neutron & FNAL W-asymmetry data



Accardi et al. (2016)



→ extrapolated ratio at $x = 1$

$$d/u \;\rightarrow\; 0.09 \pm 0.03$$

→ upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine d/u up to $x \sim 0.85$

Outlook

- Confirmation of duality (experimentally & theoretically) suggests origin in dynamical cancelations between resonances

  $\longrightarrow$ explore more realistic descriptions based on phenomenological $\gamma^* N N^*$ form factors

  $\longrightarrow$ incorporate nonresonant background in same framework

- Practical application of duality

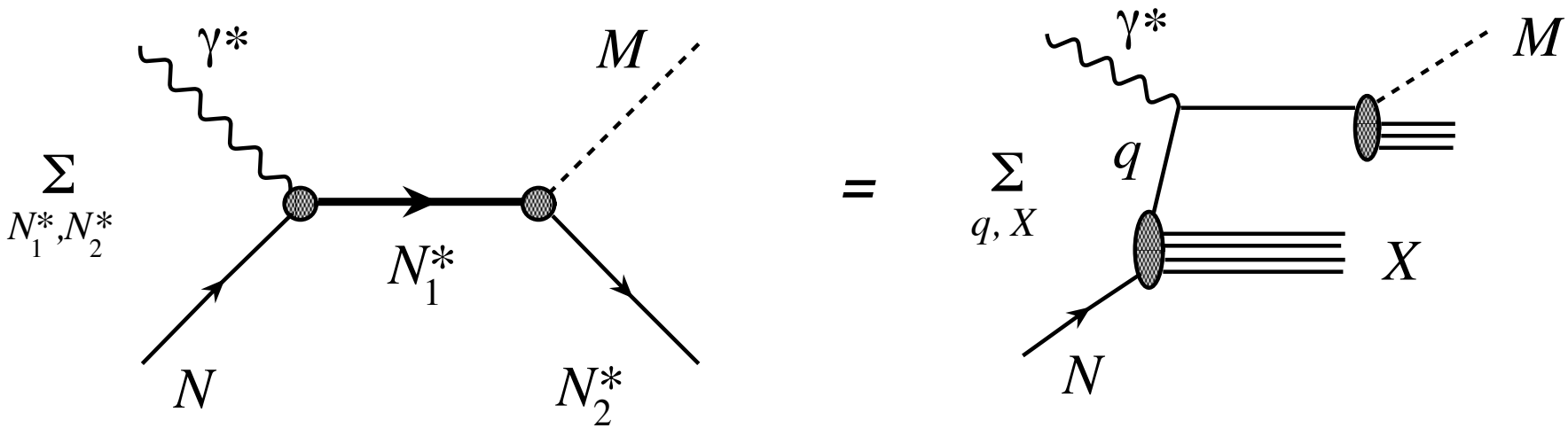
  $\longrightarrow$ use resonance region data to constrain PDFs at high x

- Extend quark-hadron duality concept to $e.g.$ electroproduction

  $\longrightarrow$ application to semi-inclusive DIS, DVCS / GPDs, ...

Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction



s-channel resonance excitation and decay

parton level scattering and fragmentation

$$\sum_{N_2^*} \left| \sum_{N_1^*} F_{\gamma N \to N_1^*}(Q^2, M_1^*)\, \mathcal{D}_{N_1^* \to N_2^* M}(M_1^*, M_2^*) \right|^2 = \sum_q e_q^2\, q(x, Q^2)\, D_q^M(z, Q^2)$$

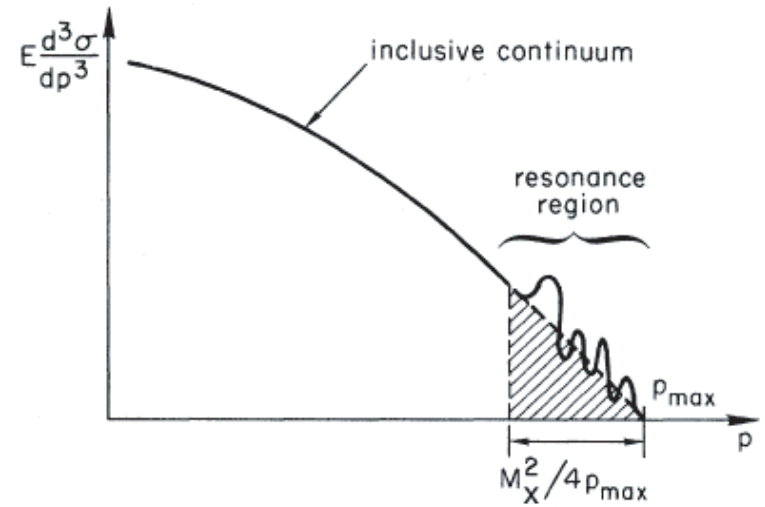
Duality in exclusive reactions

- Exclusive–inclusive correspondence principle:

  → continuity of dynamics from one (known) region to another (poorly known)

$$\int_{p_{\max}-M_X^2/4p_{\max}}^{p_{\max}} dp \;\; E\frac{d^3\sigma}{dp^3}\bigg|_{\rm incl} \sim \sum_{\rm res} E\frac{d\sigma}{dp_T^2}\bigg|_{\rm excl}$$

$$\gamma^* N \to M\ X \qquad\qquad \gamma^* N \to M\ N^*$$



→ resonance contribution to $d\sigma$ should be comparable to the continuum contribution extrapolated from high energy

$$\frac{E}{\sigma}\frac{d^3\sigma}{dp^3} \equiv f(x, p_T^2, sQ^2) \longrightarrow f(x, p_T^2, sQ^2) \overset{s\to\infty}{\longrightarrow} f(x, p_T^2)$$