Connecting resonance physics to deep-inelastic phenomena: studies in quark-hadron duality

Wally Melnitchouk
Operated by the Southeastern Universities Research Association for the U.S. Department of Energy

Thomas Jefferson National Accelerator Facility

Page 16

Pentaquark Summary

• Existence or otherwise is a CRUCIAL question in strong interaction physics
• Wilczek, Jaffe: That we cannot say whether such exotica exist or not shows HOW LITTLE WE UNDERSTAND NON-PERTURBATIVE QCD
• Jefferson Lab is the ideal facility to definitively answer this question!

Duality hypothesis: complementarity between quark and hadron descriptions of observables

\[ \sum_{\text{hadrons}} = \sum_{\text{quarks}} \]

→ can use either set of complete basis states to describe physical phenomena
In practice, at finite energy typically have access only to limited set of basis states.

Question is not why duality exists, but how it arises where it exists, and how can we make use of it?
Duality in hadron-hadron scattering

Igi (1962)
Dolen, Horn, Schmidt (1968)

\[ \sigma^{\pi^+ p} - \sigma^{\pi^- p} \]

\[ p_{\text{Lab}} \Delta \sigma \]

(mb GeV)

\[ p_{\text{Lab}} \text{ (GeV/c)} \]

\[ \sum_{R} \alpha_{j}(t) \]

"s-t channel duality"
Duality in electron-nucleon scattering

“Bloom-Gilman duality”

\[
\frac{2M}{Q^2} \int_0^{\nu_m} d\nu \; \nu W_2(\nu, Q^2) = \int_1^{\omega'_m} d\omega' \; \nu W_2(\omega')
\]

“finite-energy sum rules”
Duality in electron-nucleon scattering

\[ \xi = \frac{2x}{1 + \frac{\sqrt{1 + 4M^2x^2}}{Q^2}} \]

average over resonances 
(strongly \( Q^2 \) dependent)

\[ \approx \quad Q^2 \text{ independent scaling function} \]
Duality in electron-nucleon scattering

- **In deep-inelastic region** \((W \gtrsim 2 \text{ GeV}, \; Q^2 \gtrsim 1 \text{ GeV}^2)\)
structure function given by parton distributions

\[
F_2(x, Q^2) \overset{\text{lo}}{=} x \sum_q e_q^2 \; q(x, Q^2)
\]

- **In resonance region** \((W \lesssim 2 \text{ GeV})\), or at low \(Q^2\) \((Q^2 \lesssim 1 \text{ GeV}^2)\)
can no longer resolve individual quark structure

- Resonance and DIS regions intimately connected
  → resonances an integral part of scaling structure function
  
  *e.g.* in large-\(N_c\) limit, spectrum of zero-width resonances is “maximally dual” to quark-level (smooth) structure function
Scaling functions from resonances

Earliest attempts predate QCD

- e.g. harmonic oscillator spectrum \( M_n^2 = (n + 1)\Lambda^2 \)
  including states with spin = 1/2, ..., \( n+1/2 \)
  \( (n \text{ even}: I = 1/2, \ n \text{ odd}: I = 3/2) \)  
  \[ \text{Domokos et al. (1971)} \]

- at large \( Q^2 \) magnetic coupling dominates
  \[ G_n(Q^2) = \frac{\mu_n}{(1 + Q^2r^2/M_n^2)^2} \]
  \( r^2 \approx 1.41 \)

- in Bjorken limit, \( \sum_n \longrightarrow \int dz \), \( z \equiv M_n^2/Q^2 \)
  \[ F_2 \sim (\omega' - 1)^{1/2}(\mu_{1/2}^2 + \mu_{3/2}^2) \int_0^\infty dz \frac{z^{3/2}(1 + r^2/z)^{-4}}{z + 1 - \omega' + \Gamma_0^2z^2} \]

- scaling function of \( \omega' = \omega + M^2/Q^2 \) \( (\omega = 1/x) \)
Scaling functions from resonances

- **Earliest attempts predate QCD**
  
  \[ M_n^2 = (n + 1)\Lambda^2 \]
  
  including states with spin = 1/2, ..., n+1/2
  
  \( (n \text{ even: } I = 1/2, \quad n \text{ odd: } I = 3/2) \)
  
  \[ F_2 \sim (\mu_{1/2}^2 + \mu_{3/2}^2) \frac{(\omega' - 1)^3}{(\omega' - 1 + r^2)^4} \]

  \( \text{cf. Drell-Yan-West relation} \)

  \[ G(Q^2) \sim \left( \frac{1}{Q^2} \right)^m \quad \iff \quad F_2(x) \sim (1 - x)^{2m-1} \]

  \( \rightarrow \) similar behavior found in many models

  \( \text{Einhorn (1976) ('}t\text{ Hooft model)} \)

  \( \text{Greenberg (1993) (NR scalar quarks in HO potential)} \)

  \( \text{Pace, Salme, } \text{Lev (1995) (relativistic HO with spin)} \)

  \( \text{Isgur et al. (2001) (transition to scaling)} \)

  \( \ldots \)
Scaling functions from resonances

Phenomenological analyses at finite $Q^2$

→ additional constraints from threshold behavior at $q \to 0$
and asymptotic behavior at $Q^2 \to \infty$

$$
\left(1 + \frac{\nu^2}{Q^2}\right) F_2^R = M \nu \left[ |G_+|^2 + 2|G_0|^2 + |G_-|^2 \right] \delta(W^2 - M_R^2)
$$

Davidovsky, Struminsky (2003)

→ 21 isospin-1/2 & 3/2 resonances (with mass < 2 GeV)

$$
|G_\pm(Q^2)|^2 = |G_\pm(0)|^2 \left( \frac{|q|}{|q_0|} \frac{\Lambda^2}{Q^2 + \Lambda'^2} \right)^{\gamma_1} \left( \frac{\Lambda^2}{Q^2 + \Lambda^2} \right)^{m_\pm}
$$

$m_{+,0,-} = 3, 4, 5$

$$
|G_0(Q^2)|^2 = C^2 \left( \frac{Q^2}{Q^2 + \Lambda''^2} \right)^{2a} \frac{q_0^2}{|q|^2} \left( \frac{|q|}{|q_0|} \frac{\Lambda^2}{Q^2 + \Lambda^2} \right)^{\gamma_2} \left( \frac{\Lambda^2}{Q^2 + \Lambda^2} \right)^{m_0}
$$

→ in $x \to 1$ limit,

$$
F_2(x) \sim (1 - x)^{m_+}
$$
Scaling functions from resonances

Phenomenological analyses at finite $Q^2$

--- valence-like structure of dual function suggests “two-component duality”:

- **valence** (Reggeon exchange) dual to **resonances** $F_{2}^{(\text{val})} \sim x^{0.5}$
- **sea** (Pomeron exchange) dual to **background** $F_{2}^{(\text{sea})} \sim x^{-0.08}$
Scaling functions from resonances

- Explicit realization of Veneziano & Bloom-Gilman duality

\[ X^2 = \sum_X X = \sum_R R = \sum_{\text{Res}} \text{Res} \]

- Veneziano model not unitary, has no imaginary parts

- Generalization of narrow-resonance approximation, with nonlinear, complex Regge trajectories

\[ D(s, t) = \int_0^1 dz \left( \frac{z}{g} \right)^{-\alpha_s(s(1-z))^{-1}} \left( \frac{1-z}{g} \right)^{-\alpha_t(tz)^{-1}} \]

"dual amplitude with Mandelstam analyticity" (DAMA) model

Jenkovszky et al.
Scaling functions from resonances

- **Explicit realization of Veneziano & Bloom-Gilman duality**

  for large $x$ and $Q^2$, have power-law behavior

  \[ F_2 \sim (1 - x)^{2\alpha_t(0)} \ln 2g / \ln g \]

  where parameter $g$ can be $Q^2$ dependent

---

Jenkovszky, Magas, Londergan, Szczepaniak (2012)
Duality and QCD

- **Operator product expansion**

  -> **expand moments of structure functions in powers of** $1/Q^2$

  \[ M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) \]

  \[ = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots \]

  matrix elements of operators with specific “twist” $\tau$

  \[ \tau = \text{dimension} - \text{spin} \]

  $\tau = 2$  \hspace{1cm} $\tau > 2$
Duality and QCD

- **Operator product expansion**
  
  → expand *moments* of structure functions in powers of $1/Q$,

  \[
  M_n(Q^2) = \int_0^1 dx \ x^{n-2} \ F_2(x, Q^2) 
  = A_n^{(2)} + \frac{A_n^{(4)}}{Q^2} + \frac{A_n^{(6)}}{Q^4} + \cdots
  \]

- If moment $\approx$ independent of $Q^2$
  
  → “higher twist” terms $A_n^{(\tau > 2)}$ small

- Duality $\leftrightarrow$ suppression of higher twists
Truncated moments of $F_2^p$ in resonance region

$\int \frac{f}{f_0} \, dx$ for $Q^2 > 1 \text{ GeV}^2$

$\rightarrow$ higher twists $< 10\text{–}15\%$ for $Q^2 > 1 \text{ GeV}^2$

Malace et al. (2009)
Resonances & twists

- Total “higher twist” is *small* at scales $Q^2 \sim \mathcal{O}(1 \text{ GeV}^2)$
- On average, nonperturbative interactions between quarks and gluons not dominant (at these scales)
  $\rightarrow$ nontrivial interference between resonances

---

- Can we understand this dynamically, at quark level?
  $\rightarrow$ is duality an accident?

- Can we use resonance region data to learn about leading *twist* structure functions (and *vice versa*)?
  $\rightarrow$ expanded data set has potentially significant implications for global quark distribution studies
Consider simple quark model with spin-flavor symmetric wave function

\[ d\sigma \sim \left( \sum_i e_i \right)^2 \]

For duality to work, these must be equal

\[ d\sigma \sim \sum_i e_i^2 \]

how can \textit{square of a sum} become \textit{sum of squares}?
Dynamical cancellations

→ e.g. for toy model of two quarks bound in a harmonic oscillator potential, structure function given by

\[ F(\nu, q^2) \sim \sum_n |G_{0,n}(q^2)|^2 \delta(E_n - E_0 - \nu) \]

→ charge operator \( \sum_i e_i \exp(iq \cdot r_i) \) excites

\begin{align*}
\text{even partial waves with strength } & \propto (e_1 + e_2)^2 \\
\text{odd partial waves with strength } & \propto (e_1 - e_2)^2
\end{align*}

→ resulting structure function

\[ F(\nu, q^2) \sim \sum_n \{ (e_1 + e_2)^2 G_{0,2n}^2 + (e_1 - e_2)^2 G_{0,2n+1}^2 \} \]

→ if states degenerate, cross terms \( \sim e_1 e_2 \) cancel when averaged over nearby even and odd parity states

Close, Isgur (2001)
Dynamical cancellations

- duality is realized by summing over at least one complete set of even and odd parity resonances

- in NR Quark Model, even & odd parity states generalize to 56 (L=0) and 70 (L=1) multiplets of spin-flavor SU(6)

<table>
<thead>
<tr>
<th>representation</th>
<th>$^2$8[56$^+$]</th>
<th>$^4$10[56$^+$]</th>
<th>$^2$8[70$^-$]</th>
<th>$^4$8[70$^-$]</th>
<th>$^2$10[70$^-$]</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1^p$</td>
<td>$9\rho^2$</td>
<td>$8\lambda^2$</td>
<td>$9\rho^2$</td>
<td>0</td>
<td>$\lambda^2$</td>
<td>$18\rho^2 + 9\lambda^2$</td>
</tr>
<tr>
<td>$F_1^n$</td>
<td>$(3\rho + \lambda)^2/4$</td>
<td>$8\lambda^2$</td>
<td>$(3\rho - \lambda)^2/4$</td>
<td>$4\lambda^2$</td>
<td>$\lambda^2$</td>
<td>$(9\rho^2 + 27\lambda^2)/2$</td>
</tr>
</tbody>
</table>

$\lambda$ ($\rho$) = (anti) symmetric component of ground state wave function

Close, WM (2003, 2009)
Accidental cancellations of charges?

**cat’s ears diagram**  \((4\text{-}fermion \text{ higher twist} \sim 1/Q^2)\)

\[ \propto \sum_{i \neq j} e_i e_j \sim \left( \sum_i e_i \right)^2 - \sum_i e_i^2 \]

**proton**  \(HT \sim 1 - \left(2 \times \frac{4}{9} + \frac{1}{9}\right) = 0!\)

**neutron**  \(HT \sim 0 - \left(\frac{4}{9} + 2 \times \frac{1}{9}\right) \neq 0\)

\[\rightarrow\] duality in proton a **coincidence**!

\[\rightarrow\] should **not** hold for neutron !!

*Brodsky (2000)*
Neutron: the smoking gun

- Duality in \textit{neutron} more difficult to test because of absence of free neutron targets
- New extraction method (using iterative procedure for solving integral convolution equations) allowed first determination of $F_2^n$ in resonance region & test of neutron duality
Neutron: the smoking gun

→ “theory”: global QCD fit to $W > 2$ GeV data

→ locally, violations of duality in resonance regions < 15–20% (largest in Δ region)

→ globally, violations < 10%

→ duality is not accidental, but a general feature of resonance-scaling transition!

→ use resonance region data to learn about leading twist structure functions?
Global QCD analysis of high-energy scattering data, including large-$x$, low-$Q^2$ region

Systematically study effects of $Q^2$ & $W$ cuts

- **cut0**: $Q^2 > 4 \text{ GeV}^2$, $W^2 > 12.25 \text{ GeV}^2$
- **cut1**: $Q^2 > 3 \text{ GeV}^2$, $W^2 > 8 \text{ GeV}^2$
- **cut2**: $Q^2 > 2 \text{ GeV}^2$, $W^2 > 4 \text{ GeV}^2$
- **cut3**: $Q^2 > m_c^2$, $W^2 > 3 \text{ GeV}^2$

- Larger database with weaker cuts gives significantly reduced errors, especially at large $x$
- Up to $\sim 40$–$60\%$ error reduction when cuts extended into near-resonance region
CTEQ-JLab (CJ) global PDF analysis

**Valence \(d/u\) ratio at high \(x\)**

- significant reduction of PDF errors with new JLab tagged neutron & FNAL \(W\)-asymmetry data

\[
\begin{align*}
\text{extrapolated ratio at } x &= 1 \\
\frac{d}{u} &\rightarrow 0.09 \pm 0.03
\end{align*}
\]

- upcoming experiments at JLab (MARATHON, BONuS, SoLID) will determine \(d/u\) up to \(x \sim 0.85\)

Accardi et al. (2016)
Outlook

- **Confirmation of duality** (experimentally & theoretically) suggests origin in dynamical cancelations between resonances
  - explore more realistic descriptions based on phenomenological $\gamma^* NN^*$ form factors
  - incorporate nonresonant background in same framework

- **Practical application of duality**
  - use resonance region data to constrain PDFs at high $x$

- **Extend quark-hadron duality concept to e.g. electroproduction**
  - application to semi-inclusive DIS, DVCS / GPDs, ...
Duality in (semi-inclusive) meson production

- Extend duality to less inclusive processes, such as meson electroproduction

\[ \sum_{N_1^*, N_2^*} \gamma^* N \rightarrow N_1^* q, X N_2^* \]

\[ s\text{-channel resonance excitation and decay} \]

\[ \sum_{N_1^*} \left| \sum_{N_2^*} F_{\gamma N \rightarrow N_1^*} (Q^2, M_1^*) D_{N_1^* \rightarrow N_2^* M} (M_1^*, M_2^*) \right|^2 = \sum_{q} e_q^2 q(x, Q^2) D_q^M (z, Q^2) \]

\[ \text{parton level scattering and fragmentation} \]
Duality in exclusive reactions

- **Exclusive–inclusive correspondence principle:**
  - Continuity of dynamics from one (known) region to another (poorly known)

\[
\int_{p_{\text{max}}-M_X^2/4p_{\text{max}}}^{p_{\text{max}}} dp \quad E \frac{d^3 \sigma}{dp^3} \bigg|_{\text{incl}} \sim \sum_{\text{res}} E \frac{d\sigma}{dp_T^2} \bigg|_{\text{excl}}
\]

\[
\gamma^* N \rightarrow M \ X \quad \gamma^* N \rightarrow M \ N^*
\]

- Resonance contribution to \(d\sigma\) should be comparable to the continuum contribution extrapolated from high energy

\[
E \frac{d^3 \sigma}{\sigma \, dp^3} \equiv f(x, p_T^2, sQ^2) \quad \rightarrow \quad f(x, p_T^2, sQ^2) \xrightarrow{s \to \infty} f(x, p_T^2)
\]